USING MAIMONIDES' RULE TO ESTIMATE THE EFFECT OF CLASS SIZE ON SCHOLASTIC ACHIEVEMENT

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1. Introduction

When asked about their views on class size in surveys, parents and teachers generally report that they prefer smaller classes. This may be because those involved with teaching believe that smaller classes promote student learning, or simply because smaller classes offer a more pleasant environment for the pupils and teachers who are in them.

Causal effects of class size on pupil achievement have proved very difficult to measure. Even though the level of educational inputs differs substantially both between and within schools, these differences are often associated with factors such as remedial training or students' socioeconomic background.

The twelfth century Rabbinic scholar, Maimonides, interprets the Talmud's discussion of class size as follows: ''Twenty-five children may be put in charge of one teacher. If the number in the class exceeds twenty-five but is not more than forty, he should have an assistant to help with the instruction. If there are more than forty, two teachers must be appointed''.

The importance of Maimonides' rule is that it has been used to determine the division of enrolment cohorts into classes in Israeli public schools. The maximum of 40 is well-known to school teachers and principals, and it is circulated annually in a set of standing orders from the Director General of the Education Ministry.

In this paper the class-size function induced by Maimonides' rule is used to construct instrumental variables estimates of class-size effects on pupil achievement.

This sort of identification argument has a long history in social science and can be viewed as an application of Campbell's [1969] regression-discontinuity design for evaluation research to the class size question.

2. Data and descriptive evidence

Our analysis began by linking average math and reading scores for each class with data on school characteristics and class size from other sources.

The linked data sets contain information on the population of schools covered by the Central Bureau of Statistics Censuses of Schools. These are annual reports on all educational institutions at the beginning of the school year (in September).

The data on class size are from an administrative source, and were collected between March and June of the school year that began in the previous September. The unit of observation in the linked data sets and for our statistical analysis is the class.

The linked class-level data sets include information on:

- average test scores in each class
- the spring class size
- beginning-of-the-year enrollment in the school for each grade
- a town identifier
- a school-level index of students' socioeconomic status
- variables identifying the ethnic character (Jewish/Arab) and religious affiliation (religious/secular) of schools.

The class-size function derived from Maimonides' rule can be stated formally as follows. Let e_s denote beginning-of-the-year enrollment in school s in a given grade, and let f_{sc} denote the class size assigned to class c in school s, for that grade. Assuming that cohorts are divided into classes of equal size, we have

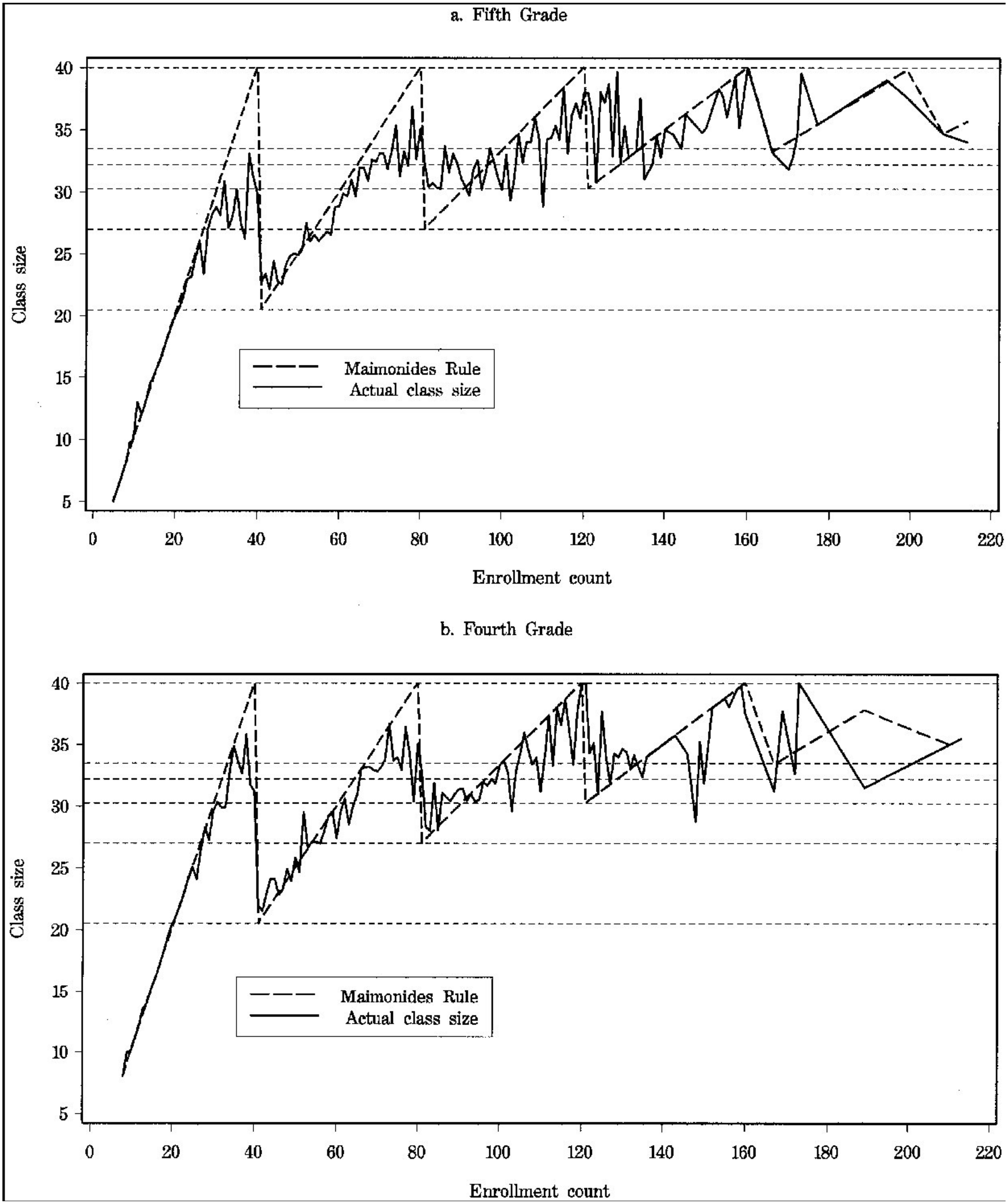
$$f_{sc} = e_s / [\mathrm{int}\,((e_s - 1)/40) + 1], \qquad (1)$$

where, for any positive number n, the function int (n) is the largest integer less than or equal to n.

Equation (1) captures the fact that Maimonides' rule allows enrollment cohorts of 1–40 to be grouped in a single class, but enrollment cohorts of 41–80 are split into two classes of average size 20.5–40, enrollment cohorts of 81–120 are split into three classes of average size 27–40, and so on.

Even though the actual relationship between class size and enrollment size involves many factors, in Israel it clearly has a lot to do with f_{sc}. This can be seen in Figures Ia and Ib, which plot the average class size by enrollment size for fifth and fourth grade pupils, along with the class-size function.

The figures show that at enrollment levels that are not integer multiples of 40, class size increases approximately linearly with enrollment size. But average class size drops sharply at integer multiples of 40, i.e., at the corners of the class size function.

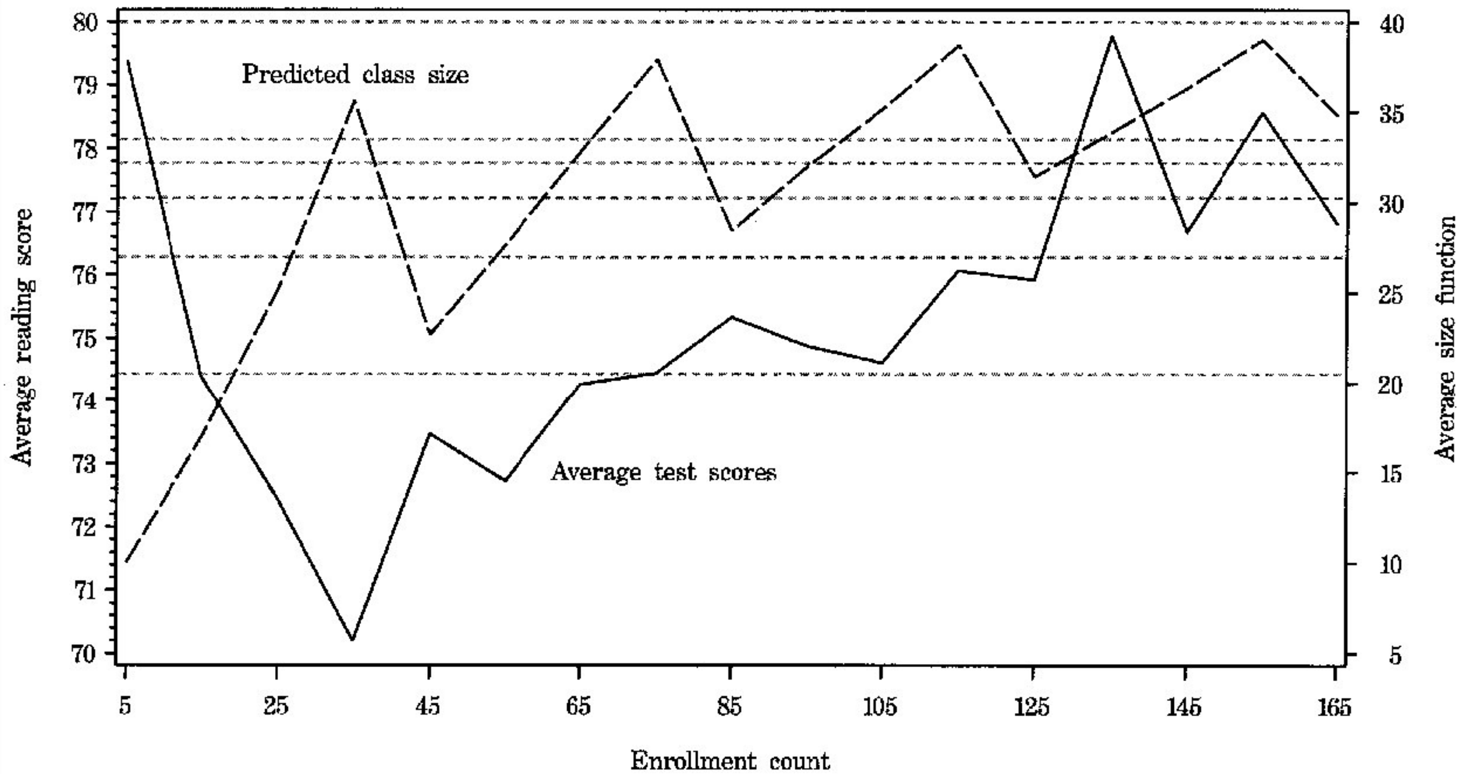

a. Fifth Grade
Class size
Enrollment count
Maimonides Rule
Actual class size
b. Fourth Grade
Class size
Enrollment count
Maimonides Rule
Actual class size

In addition to exhibiting a strong association with average class size, the class-size function is also correlated with the average test scores of fourth and fifth graders (although not third graders). This can be seen in Figures IIa and IIb, which plot average reading test scores and average values of f_{sc} by enrolment size, in enrollment intervals of ten.

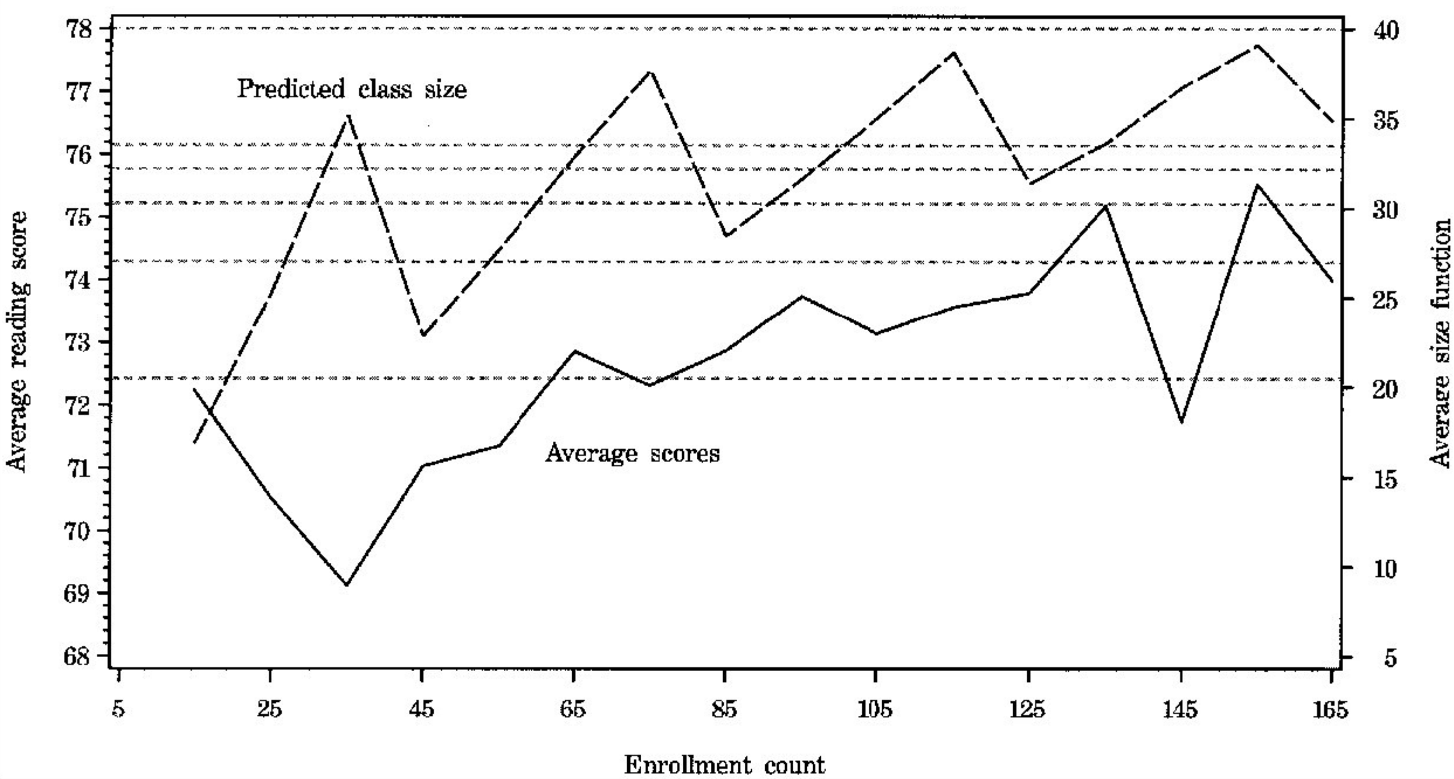
The figures show that test scores are generally higher in schools with larger enrollments and, therefore, larger predicted class sizes. Most importantly, however, average scores by enrolment size can be seen to exhibit an up-and-down pattern that is, at least in part, the mirror image of the class-size function.

The overall positive correlation between scores and enrolment is partly attributable to that fact that larger schools in Israel are more likely to be located in relatively prosperous big cities, while smaller schools are more likely to be located in relatively poor ‘‘development towns’’ outside of major urban centers. In fact, enrollment size and the PD index measuring the proportion of students who come from a disadvantaged background are highly negatively correlated.

a. Fifth Grade



b. Fourth Grade



3. Estimation of the causal effect of class size on pupil achievement.

For the ith student in class c and school s, we can write

$$y_{isc} = X_s'\beta + n_{sc}\alpha + \mu_c + \eta_s + \varepsilon_{isc}, \tag{2}$$

where y_{isc} is pupil i's score, X_s is a vector of school characteristics including enrollment, and n_{sc} is the size of class c in school s. The term μ_c is an i.i.d. random class component. The term η_s is an i.i.d. random school component. The remaining error component ε_{isc} is specific to pupils. The first two error components are introduced to parameterize possible within school and within-class correlation in scores. The class-size coefficient α is the parameter of primary interest.

Since n_{sc} is not randomly assigned, in practice it is likely to be correlated with potential outcomes (in this case, the error components in (2)). Thus, OLS estimates of (2) do not have a causal interpretation, although instrumental variables estimates still might. The causal interpretation of instrumental variables estimates turns on whether it is reasonable to assume that, after controlling for X_s, the only reason for any association between instruments and test scores is the association between instruments and class size.

The identification approach taken here exploits the fact that the regressor of interest - class size - is partly determined by a known discontinuous function of an observed covariate - enrollment).

The up-and-down pattern in the conditional expectation of test scores given enrolment is interpreted as reflecting the causal effect of changes in class size that are induced by changes in enrolment. This interpretation is plausible because the class-size function is known to share this pattern, while it seems likely that any other mechanism linking enrollment and test scores will be much smoother.

The identifying assumptions that lay behind this approach can be expressed formally by introducing some notation for the ‘‘first-stage’’ relationship of interest:

$$n_{sc} = X_s\, \pi_0 + f_{sc}\, \pi_1 + \xi_{sc}, \tag{4}$$

where π_0 and π_1 are parameters and, as before, X_s is a vector of school-level covariates that includes functions of enrollment, e_s, and measures of pupil socioeconomic status. The error term ξ_{sc} is defined as the residual from the population regression of n_{sc} on X_s and the instrument, f_{sc}. This residual captures other factors that are correlated with enrollment. These factors are probably also related to pupil achievement, which is why OLS estimates of (3) do not have a causal interpretation.

Since f_{sc} is a deterministic function of e_s, and e_s is almost certainly related to pupil test scores for reasons other than effects of changing class size, the key identifying assumption that underlies estimation using fsc as an instrument is that any other effects of e_s on test scores are adequately controlled by the terms in $X_s\beta$ in (3), and ‘‘partialled out’’ of the instrument by the term $X_s\, \pi_0$ in equation (4).

3.1. Things would go badly if….

…. parents selectively exploit Maimonides' rule so as to place their children in schools with small classes.

Selective manipulation could occur if more-educated parents successfully place children in schools with grade enrollments of 41–45, knowing that this will lead to smaller classes in a particular grade. In practice, however, there is no way to know whether a predicted (by the parents) enrollment of 41 will not decline to 38 by the time school starts, obviating the need for two small classes in the relevant grade. And even if there was a way to predict this accurately, we noted earlier that parents are not free to transfer children from one elementary school to another except by moving.

Of course, parents who discover they got a bad draw in the ''enrollment lottery'' (e.g., enrollment of 38 instead of 41) might then elect to pull their kids out of the public school system entirely. But private elementary schooling is rare in Israel outside of the ultra-orthodox communities……

4. Estimation results

To interpret results, test score is in the range 1-100.

OLS estimates with no control variables show a strong positive correlation between class size and achievement. Controlling for PD, however, the positive association largely disappears and, in some cases, becomes negative (table II).

Instrumental variables estimates for fifth graders are reported in Table IV. The instrumental variables estimate of the effect of class size on the reading scores of fifth graders in a model without any controls for enrollment size is -.16 with a standard error of .04. The estimates (standard errors) from models including linear and quadratic controls for enrollment size, reported in columns (2)–(3), range from -.26 (.08) to -.28 (.07).

Without enrolment controls, the instrumental variables estimate for fifth grade math scores is virtually zero. But in models with linear and quadratic enrollment controls, the instrumental variables estimates for the math scores of fifth graders are similar to the estimates in the corresponding models for reading scores. For example, the estimated class-size effect on math scores from a model with linear controls, reported in column (8), is -.23.

In the end, the causal effect of class size on test score is in the order of magnitude -.3/-.4: 3/4 points larger if the class is 10 pupils smaller.

TABLE II

OLS ESTIMATES FOR 1991

	5th Grade						4th Grade					
	Reading comprehension			Math			Reading comprehension			Math		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Mean score		74.3			67.3			72.5			69.9	
(s.d.)		(8.1)			(9.9)			(8.0)			(8.8)	
Regressors												
Class size	.221	-.031	-.025	.322	.076	.019	.141	-.053	-.040	.221	.055	009
	(.031)	(.026)	(.031)	(.039)	(.036)	(.044)	(.033)	(.028)	(.033)	(.036)	(.033)	(.039)
Percent disadvantaged		-.350	-.351		-.340	-.332		-.339	-.341		-.289	-.281
		(.012)	(.013)		(.018)	(.018)		(.013)	(.014)		(.016)	(.016)
Enrollment			-.002			.017			-.004			014
			(.006)			(.009)			(.007)			(.008)

The unit of observation is the average score in the class. Standard errors are reported in parentheses. Standard errors were corrected for within-school correlation between classes.

TABLE IV
2SLS ESTIMATES FOR 1991 (FIFTH GRADERS)

	Reading comprehension						Math					
	Full sample				+/- 5 Discontinuity sample		Full sample				+/- 5 Discontinuity sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Regressors												
Class size	-.158	-.275	-.260	-.186	-.410	-.582	-.013	-.230	-.261	-.202	-.185	-.443
	(.040)	(.066)	(.081)	(.104)	(.113)	(.181)	(.056)	(.092)	(.113)	(.131)	(.151)	(.236)
Percent disadvantaged	-.372	-.369	-.369		-.477	-.461	-.355	-.350	-.350		-.459	-.435
	(.014)	(.014)	(.013)		(.037)	(.037)	(.019)	(.019)	(.019)		(.049)	(.049)
Enrollment		.022	.012			.053		.041	.062			.079
		(.009)	(.026)			(.028)		(.012)	(.037)			(.036)
Enrollment squared/100			.005						-.010			
			(.011)						(.016)			

The unit of observation is the average score in the class. Standard errors are reported in parentheses. Standard errors were corrected for within-school correlation between classes.

TABLE V
2SLS ESTIMATES FOR 1991 (FOURTH GRADERS)

	Reading comprehension						Math					
	Full sample				+/- 5 Discontinuity sample		Full sample				+/- 5 Discontinuity sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Regressors												
Class size	-.110	-.133	-.074	-.147	-.098	-.150	.049	-.050	-.033	-.098	.095	.023
	(.040)	(.059)	(.067)	(.084)	(.090)	(.128)	(.048)	(.070)	(.081)	(.092)	(.114)	(.160)
Percent disadvantaged	-.346	-.345	-.346		-.354	-.347	-.290	-.284	-.284		-.299	-.290
	(.014)	(.014)	(.014)		(.034)	(.034)	(.017)	(.017)	(.017)		(.042)	(.043)
Enrollment		.005	-.040			.017		-.020	.007			.023
		(.008)	(.024)			(.022)		(.010)	(.029)			(.028)
Enrollment squared/100			.021						.006			
			(.011)						(.014)			

The unit of observation is the average score in the class. Standard errors are reported in parentheses. Standard errors were corrected for within-school correlation between classes.