

Noon Seminar # 6 - AK model

Problem 1 - Utility Enhancing Government Expenditure.

Consider the growth model where firms have the linear production function $y = Ak$ where $A > 0$ is capital per capita and y is output per capita (for simplicity there is no population growth and the size of the population is equal to one). Assume further that the government expenditure g , which is a fixed fraction of output τ , i.e. $g = \tau y = \tau Ak$, contribute to the welfare of the private agent. The government expenditure is financed by lump-sum tax T . Thus infinitely lived household maximizes the following utility

$$\int_0^\infty \frac{(c_t g_t^\alpha)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

with $\alpha > 0$.

1. Assuming perfect competition framework in production factor markets derive the expression for the interest rate and wage rate. Assume for simplicity that the rate of capital depreciation is equal to zero
2. Set up the representative household's optimisation problem. What are the state and control variables? What is the **current-value** Hamiltonian?
3. Derive the F.O.C's.
4. Derive the Euler equation. What is the growth rate of consumption at the steady state? What is the steady state growth rate of capital?
5. Set up a social planner problem. What are the control variables? Discuss possible sources of inefficiency of the decentralized equilibrium in this model.

Problem 2 - Effects of changes of parameters in Ramsey model.

Consider economy with knowledge spillovers where the representative household maximizes its lifetime welfare subject to its flow budget constraint and No-Ponzi-Game condition. Assume that the utility function is of CRRA type and that the size of the household grows at the rate $n > 0$. There is a continuum of identical perfectly competitive firms of mass one. The firm i has the production function $Y_i = AK_i^\alpha L_i^{1-\alpha} K^\alpha$ where $0 < \alpha < 1$ and $K = \int_0^1 K_i di$ is the aggregate capital. Assume further that there is a government which takes household consumption and labor income at constant rates τ_c and τ_w , respectively, and that the firms pay taxes from renting capital at constant rate τ_r . Let the government consume the amount of the tax revenues from taxing consumption and the rest is returned back to the households in the form of lump-sum transfers.

- Write down the household's problem, firm's problem and government budget constraints
- Solve for the growth rates of consumption, capital and output along the balanced growth path.
- Which taxes can affect the long run growth rate and which cannot? Explain.