JEM004 Macroeconomics
IES, Fall 2008
Seminar - set up
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Noon Seminar # 4 - Ramsey Model

Problem 1 - Effects of changes of parameters in Ramsey model.

Describe how each of the following affect the $\dot{k} = 0$ and $\dot{c} = 0$ locus. How does c and k react immediately after change and what is the new steady state?

a) Permanent fall in productivity growth rate g

b) A rise in the preference for today's consumption θ

c) Proportional downward shift of production function $f(k)$

d) Increase in the depreciation rate of capital from 0 to positive δ

Problem 2 - Ramsey model with government purchases in utility function.

Consider an economy a la Ramsey with infinitely lived representative households, who provide labor services in exchange for wages, receive interest income on assets, purchase goods for consumption and save by accumulating additional assets. We will modify here the standard RCK model by assuming that government purchases affect utility from private consumption and that government purchases and private consumption are perfect substitutes. Thus the representative household maximizes its lifetime welfare

$$\int_0^\infty u(c_t, g_t)e^{-(\rho-n)t}dt = \int_0^\infty \frac{(c_t + g_t)^{1-\theta}}{1-\theta}e^{-(\rho-n)t}dt$$

subject to its flow budget constraint and the No-Ponzi-Game condition, where n is the rate of population growth, $\theta, \rho > 0$ and $\rho > n$. Assume further that the government purchases per capita $g_t = \frac{G_t}{L_t}$, which are financed by the constant tax on consumption $1 > \tau_c > 0$, are such that the government budget is balanced at any moment of time. The production sector of the economy is again according to the Ramsey model composed of representative perfectly competitive firms which produce goods, pay wages for labor input and make rental payments for capital inputs. The firms have neoclassical production function, expressed in per capita terms $y_t = Ak_t^\alpha$ where $0 < \alpha < 1$ and capital depreciates at the rate $\delta > 0$.

(a) Specify the household's dynamic optimization problem. Explain in words the meaning of the No-Ponzi-Game condition for household.

(b) Derive the first order conditions of the household's optimization problem.

(c) Write down the government's flow budget constraint. Is it in our case also necessary to specify the NPG to constraint the government behavior? Explain why or why not.

(d) Derive and explain the Euler equation.

(e) Write down and solve the problem of a profit-maximizing representative form. Using the results above specify the competitive market equilibrium.

(f) Derive the conditions for the steady-state level of capital and consumption per capita. Draw the phase diagram.

(g) Assume that the economy is initially at the steady state with $c^* > 0$. What are the effects of a temporary increase in government purchases on the paths of consumption, capital and interest rate (draw their behavior over time). How these effects will be changed if the increase in government purchases will be announced at some time T>0 before it really happens?

(h) Write down the social planner problem for this economy and derive the first order conditions. Let denote the path of optimal consumption when government spending is zero ($g_t = 0$ for all t) by $c^{0\infty}_{t\,t=0}$. Do your results support the view that any equilibrium with the time path of government spending $g_{t\,t=0}^{\infty}$ (such that $g_t \leq c_t^0$ for all t) is a social optimum? Having this in mind is the competitive market equilibrium derived earlier a social optimum? How this result will be changed if the government uses a lump sum tax instead of consumption tax to finance their purchases? Which role in this sense do you think plays the assumption of perfect substitutability between private consumption and government purchases in the model?

(i) How will be the Euler equation of the original competitive market economy above changed if the elasticity of substitution between private consumption and government purchases is equal to one, i.e. the household preferences are given by $u(c, g) = \frac{(c^\gamma g^{1-\gamma})^{1-\theta}}{1-\theta}$ where $0 < \gamma < 1$ and $\gamma(1-\theta) < 1$.