3.6 Dynamics of the Economy

Let us draw the **phase diagram** of this system of differential equations, taking into consideration transversality condition $\lim_{t\to\infty}\hat{k}e^{-\int_0^t (f'(\hat{k})-\delta-n-g)dv}=0$

$$\begin{aligned}\dot{\hat{k}} &= f(\hat{k})-(g+n+\delta)\hat{k}-\hat{c}\\ \frac{\dot{\hat{c}}}{\hat{c}} &= \frac{1}{\theta}(f'(\hat{k})-\delta-\rho-\theta g)\end{aligned}$$

For locus $\dot{\hat{k}}=0$ we get the expression for $\hat{c}$ as a function of $\hat{k}$ - $\hat{c}=f(\hat{k})-(g+n+\delta)$. From the Figure 1 we see that consumption is an increasing function of capital up to the point where $f'(\hat{k}_{GOLD})-\delta=g+n$, and then changes to decreasing function. Level of $\hat{k}_{GOLD}$ which maximizes consumption is called, similarly as in Sollow model, golden rule level of capital. For all points lying above the locus $\hat{c}>f(\hat{k})-(g+n+\delta)$ and therefore $\dot{\hat{k}}<0$, i.e. level of capital per effective worker is decreasing. Opposite is true for points lying under the locus.

Note that locus $\dot{\hat{c}}=0$ is independent of the level of $\hat{c}$, thus it directly pinpoints the equilibrium level of $\hat{k}^*$ which will have to satisfy condition $f'(\hat{k}^*)-\delta=\rho+\theta g$. Therefore, locus $\dot{\hat{c}}=0$ will be a vertical line through this level of capital. Moreover, as transversality condition implies that $f'(\hat{k}^*)-\delta>g+n$, we see that $\hat{k}^*<\hat{k}_{GOLD}$, i.e. the vertical line will be to the left of the golden rule level of capital $\hat{k}_{GOLD}$.[4] For all points lying left to the locus $\hat{k}<\hat{k}^* \Rightarrow f'(\hat{k})>f'(\hat{k}^*) \Rightarrow \dot{\hat{c}}>0$, i.e. level of consumption per effective worker is increasing. Opposite is true for points lying right to the locus.

The phase diagram of this system is depicted in the Figure 2. We see that this system of differential equations have **3 equilibria**: point 0 ($\hat{c}=0,\hat{k}=0$), point where $\hat{c}=0$ and $\hat{k}=\hat{k}^{**}$ (i.e. where we spend all output on depreciation of capital) and point $(\hat{c}^*,\hat{k}^*)$. However, we are only interested in equilibria with positive consumption $\hat{c}>0$. This equilibrium is unstable with saddle path. For further analysis of transitional dynamics, see Romer, p.60 (+ I will discuss it on the lecture).

Saddle path :

- **policy function:** for each level of capital per effective worker $\hat{k}$ there is a unique level of consumption $\hat{c}$ that is consistent with household's optimisation problem as well as law of motion for capital.
- shape depends on the parameters of the model: e.g higher θ (higher preference for today's consumption) implies that on the path to the steady state, household will have high levels of consumption but the convergence will be slower (the saddle

[4]Note that this fact has two implication for the steady state characteristics of the economy. First, there is no inefficient oversaving (like in Solow). However, optimizing households does not save enough to attain the maximum consumption.

path will be close to $\dot{\hat{k}} = 0$ locus). On the other hand, if θ is low, households will sacrifice current consumption for faster convergence to the steady state with high level of consumption in future.

3.7 Introduction of government - policy analysis

- new agent in the economy = government
- collects money = taxation
  - what to tax: labor income, consumption (VAT), capital income, profits of firms
  - how: lump sum, flat (proportional), progressive (brackets)
- spends money
  - own consumption ("overheads") + public goods (education, infrastructure) - enters households' utility = G
  - transfers (redistribution of income, e.g. retirement benefits) = V
- **Government's budget constraint** (generalized for flat rate case):

$$G + V = \tau_w wL + \tau_a r(Assets) + \tau_c C + \tau_f(firm's\ earnings)$$

- **Question:** How do government's policies (taxation / spending) affect the steady state of economy?

In all analyzed cases we assume zero technological growth and by g we denote government consumption per capita (instead of growth rate of technology). We compare the situations with the steady-state values without existence of government

3.7.1 Lump sum tax τ + nonproductive spending G

- firms' problem unchanged - determine $r = f'(k) - \delta; w = f(k) - f'(k)k$
- government's budget constraint: $G = \tau L; \quad \tau = G/L = g$
- household's budget constraint: $\dot{a} = w + ra - na - c - \tau$

Hamiltonian for household's problem:

$$H = u(c)e^{-(\rho-n)t} + \mu[w + ra - na - c - \tau]$$

- $\frac{\partial H}{\partial c}$ and $\partial H/\partial a$ do not change => Euler equation is unchanged

In the equilibrium we plug in for w, r (firm's problem) and g (gvt BC) and replace $k = a$

$$\begin{aligned}\frac{\dot{c}}{c} &= \frac{1}{\theta}[f'(k) - \delta - \rho] \\ \dot{k} &= f(k) - (n+\delta)k - c - g\end{aligned}$$

- k^* unchanged, c^* lower (exactly to offset government spending)
- Reason: lump sum tax = take part of income, decision making unchanged

3.7.2 Flat labor income tax τ_w + nonproductive spending G

- firms' problem unchanged - determine $r = f'(k) - \delta; w = f(k) - f'(k)k$
- government's budget constraint: $G = \tau_w wL; \quad \tau_w = g/w$
- household's budget constraint: $\dot{a} = (1-\tau_w)w + ra - na - c$

Hamiltonian for household's problem:

$$H = u(c)e^{-(\rho-n)t} + \mu[(1-\tau_w)w + ra - na - c]$$

- $\frac{\partial H}{\partial c}$ and $\partial H/\partial a$ do not change => Euler equation is unchanged

In the equilibrium we plug in for w, r (firm's problem) and g (gvt BC) and replace $k = a$

$$\begin{aligned}\frac{\dot{c}}{c} &= \frac{1}{\theta}[f'(k) - \delta - \rho] \\ \dot{k} &= f(k) - (n+\delta)k - c - g\end{aligned}$$

- k^* unchanged, c^* lower (exactly to offset government spending)
- Reason: inelastic supply of labor - HH cannot adjust (like lump sum tax)

3.7.3 Flat capital income tax τ_a + nonproductive spending G

- firms' problem unchanged - determine $r = f'(k) - \delta; w = f(k) - f'(k)k$
- government's budget constraint: $G = \tau_a raL; \quad \tau_a = g/(ra)$
- household's budget constraint: $\dot{a} = w + (1-\tau_a)ra - na - c$

Hamiltonian and F.O.C.'s for household's problem:

$$\begin{aligned}H &= u(c)e^{-(\rho-n)t} + \mu[w + (1-\tau_a)ra - na - c] \\ \frac{\partial H}{\partial c} &= 0: \quad u'(c)e^{-(\rho-n)t} = \mu \\ \frac{\partial H}{\partial a} &= -\dot{\mu} \quad \mu[(1-\tau_a)r - n] = \dot{\mu}\end{aligned}$$

- new Euler equation therefore looks $\frac{\dot{c}}{c} = \frac{1}{\theta}[(1-\tau_a)r - \rho]$

In the equilibrium we plug in for w, r (firm's problem) and g (gvt BC) and replace $k = a$

$$\begin{aligned}
\frac{\dot{c}}{c} &= \frac{1}{\theta}\Big[(1-\tau_a)(f'(k)-\delta) - \rho\Big] \\
\dot{k} &= f(k) - (n+\delta)k - c - g
\end{aligned}$$

- k^* lower, c^* lower
- Reason: HH's adjust accumulation of assets to keep consumption -> lower capital investment -> lower total output -> even lower consumption
- if taxation affects decision making of HH = **distortionary taxation**

3.7.4 Flat capital income tax τ_a + transfers V

- firms' problem unchanged - determine $r = f'(k) - \delta; w = f(k) - f'(k)k$
- government's budget constraint: $V = \tau_a raL; \quad \tau_a = v/(ra)$
- household's budget constraint: $\dot{a} = w + (1-\tau_a)ra - na - c + v$

Hamiltonian and F.O.C.'s for household's problem:

$$\begin{aligned}
H &= u(c)e^{-(\rho-n)t} + \mu[w + (1-\tau_a)ra - na - c + v] \\
\frac{\partial H}{\partial c} &= 0: \quad u'(c)e^{-(\rho-n)t} = \mu \\
\frac{\partial H}{\partial a} &= -\dot{\mu} \quad \mu[(1-\tau_a)r - n] = \dot{\mu}
\end{aligned}$$

- new Euler equation therefore looks $\frac{\dot{c}}{c} = \frac{1}{\theta}[(1-\tau_a)r - \rho]$

In the equilibrium we plug in for w, r (firm's problem) and v (gvt BC) and replace $k = a$

$$\begin{aligned}
\frac{\dot{c}}{c} &= \frac{1}{\theta}\Big[(1-\tau_a)(f'(k)-\delta) - \rho\Big] \\
\dot{k} &= f(k) - (n+\delta)k - c
\end{aligned}$$

- k^* lower, c^* lower
- Reason: even though taxes come back in the form of transfers, HH's still adjust accumulation of assets due to lower rate of return -> lower capital investment -> lower total output -> lower consumption
- still **distortionary taxation**

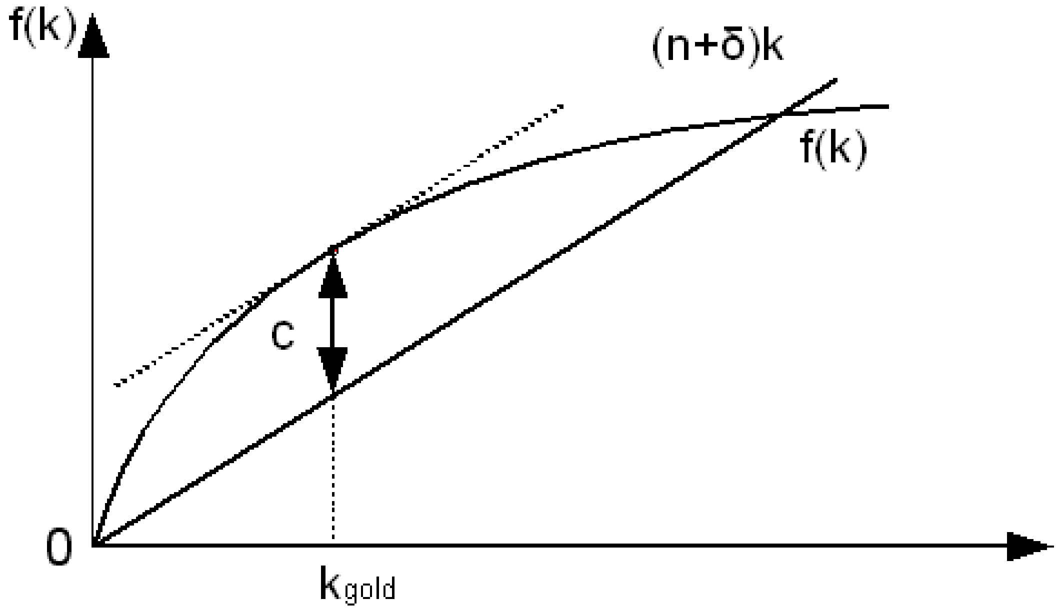

Figure 1: Consumption as a function of k - RHS of $\dot{k} = 0$ locus.

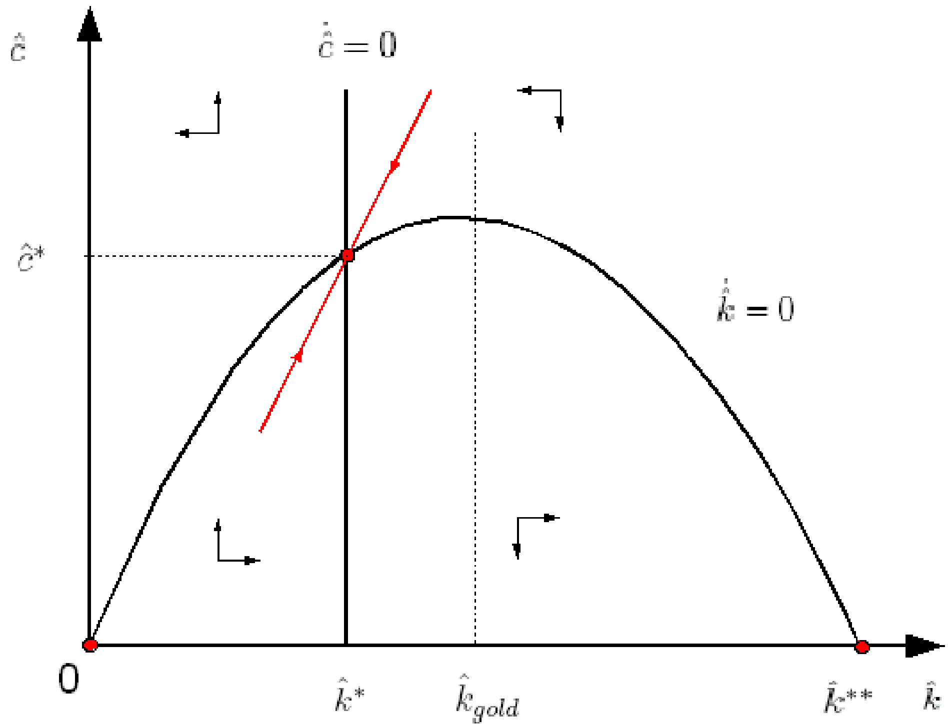

Figure 2: Phase diagram of the Ramsey model.