

Correction: On the seminar No. 2 I gave you the wrong link to the textbook in which you can find the chapter about Hamiltonians (Chapter 4.3.). Here is the correct link:

http://www.cerge-ei.cz/pdf/lecture_notes/LN01.pdf

Solow model

$$Y(t) = F(K(t), A(t)L(t)) = K(t)^\alpha (A(t)L(t))^{1-\alpha} \quad (1)$$

$$\dot{L}(t) = nL(t) \quad (2)$$

$$\dot{A}(t) = gA(t) \quad (3)$$

$$\dot{K}(t) = I(t) - \delta K(t) \quad (4)$$

$$I(t) = sF(K, L) \quad (5)$$

1. Prove that production function (1) satisfies the following properties: constant returns to scale in capital and effective labor, positive and diminishing marginal products from both production factors, essentiality condition and Inada-type conditions.

2. Express the growth rate of output in terms of growth rates of capital, labor and technology.

3. Express the production function in its intensive form (i.e. in terms of output and capital per effective unit of labor).

4. Derive the basic equation of the Solow model, i.e. show that the differential equation for the accumulation of capital per effective unit of labor has the form:

$$\dot{k}(t) = sk(t)^\alpha - (n + g + \delta)k(t) \quad (6)$$

5. Plot the Solow diagram.

6. Example from Romer (1.5.)

a) Find expressions for optimal k^* , y^* , c^* as functions of the parameters of the model - s, n, δ, g and α .

b) What is the golden-rule value of k^* ?

c) What saving rate is needed to yield the golden-rule capital stock?

7. Example from Romer (1.4.)

Consider an economy with technological progress but without population growth that is on its balanced growth path. Now suppose there is a onetime jump in the number of workers:

(a) At the time of the jump, does output per unit of effective labor rise, fall or stay the same. Why?

(b) After the initial change in output per unit of effective labor when the new workers appear, is there any further change in output per unit of effective labor? If so, does it rise or fall? Why?

(c) Once the economy has again reached a balanced growth path, is output per unit of effective labor higher, lower, or the same as it was before the new workers appeared? Why?