Seminar to Advanced Macroeconomics

Estimation of the Solow Growth Model
Economic Convergence

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Outline

- The Solow model (Empirical verification – cont.)
- Adding human capital into the Solow model
- Convergence

Last time...

- We derived the **Cross-country growth regression**

$$\log\left(\frac{Y}{L}\right) = \log A_0 + g\,t + \frac{\alpha}{1-\alpha}\log s - \frac{\alpha}{1-\alpha}\log(n+g+\delta) \qquad (1)$$

- Which can be rewritten as:

$$\log y_i = \beta_0 + \beta_1 \log s_i + \beta_2 \log(n_i + g + \delta) + \varepsilon_i \qquad (2)$$

- Equation (2) will be estimated.
- Data: 75 countries, 1960-1985 averages for data of rates, incomes for both years.
- Data in tables: for each country we have YL60; YL85 (income/labour; absolute), DY6085 (average depreciation rate), N6085 (population growth rate, average, %), IY (I/Y ~ s; average, %).

Last time commands - script

```
open /.../mrw.gdt
logs gdp85
genr s = log(inv/100)
genr ngd = log(popgrow/100+0.05)
smpl nonoil --dummy
# model 1
ols l_gdp85 const s ngd
reset
modtest --breusch-pagan
modtest --white
modtest --normality
vif
restrict
b[2]+b[3]=0
end restrict
```

Solow model – Estimation results

```
                      Ordinary least squares
    Dependent variable: ln(MacroSolow[YL85])
    Number of observations: 75
   Variable                Coefficient  St. Error   t-statistic  Sign.
1 Constant                   5.3676983   1.540081    3.4853332   [0.0008]
2 ln((MacroSolow[N6085]/100)+0.05)
                            -2.0133899   0.5328300  -3.7786717   [0.0003]
3 ln(MacroSolow[IY]/100)
                             1.3253532   0.1706108   7.7682812   [0.0000]
  R^2adj. = 59.063938603%   DW = 1.9816
  R^2 = 60.170318641%       S.E. = 0.6094559879
  Residual sum of squares:  26.7434352866204
  Maximum loglikelihood: -67.750424484184
  AIC =  1.9133446529        BIC =  2.036944019
     F(2,72) = 54.38486 [0.0000]
  Normality: Chi^2(2) = 5.81677  [0.0546]
  Heteroskedasticity: Chi^2(1) = 0.321696 [0.5706]
  Functional form: Chi^2(1) = 0.456655 [0.4992]
  AR(1) in the error: Chi^2(1) = 1.27E-04 [0.9910]
```

Solow model – Comments

- 60% of differences explained with this model!
- Diagnostics OK (variables significant, heteroscedasticity OK, normality OK, DW-statistics is OK as well, Reset test gives good results, too).
- Signs – as expected.
- However – if the specification OK, $\beta_1 = -\beta_2$.
- Values? Different: -2.013 and 1.325
- But can we reject the hypothesis $\beta_1 = -\beta_2$?

Adding human capital into Solow model

- Solow model: output determined by acumulation of capital and technological progress
- What else can improve the prediction of the model?
- Human capital: schooling and investnents into education (both private and public), into health and also opportunity costs of education.
- Production function

$$Y(t)=K(t)^{\alpha}H(t)^{\beta}(A(t)L(t))^{1-\alpha-\beta}$$

Adding human capital into Solow model

- Production function

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta}$$

- s_k and s_H

$$\dot{k}(t) = s_k y(t) - (n + g + \delta) k(t),$$
$$\dot{h}(t) = s_h y(t) - (n + g + \delta) h(t),$$

- setting both eq. to zero we get the steady state:

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)}$$

$$h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)}$$

Adding human capital into Solow model

- Production function

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta}$$

- Steady state:

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)} \qquad h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)}$$

- Inserting into production function $y = k^{\alpha} h^{\beta}$, taking logs as with the Solow model implies

$$\ln \left[\frac{Y(t)}{L(t)} \right] = \ln A(0) + gt - \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) + \frac{\alpha}{1 - \alpha - \beta} \ln(s_k) + \frac{\beta}{1 - \alpha - \beta} \ln(s_h).$$

Human capital model

- Thus:
- $$lny_i = \beta_0 + \beta_1 lns_{ki} + \beta_2 ln(n_i + g + \delta) + \beta_3 lns_{hi} + \varepsilon_i$$
- Problem: What to are the “savings on human capital”?

➔ Some proxy variable needed.

- **Proxy variable**: known, measurable and observable variable, which is correlated with the original one. Similar intuition and limitations as with instrumental variables – quality and results depend on the quality of proxy/instrument chosen.

Note to the measuring of human capital

- What proxy? Mankiw-Romer and Weil suggest the variable „SCHOOL“: enrollment rate on secondary schools in the age of 12-17 × share of population of age 15-19.
- This means how many people decided for education against working.
- s_H – savings/accumulation of human capital.
- If bad proxy – estimation results implies non-significancy of SCHOOL although possible correlation of human capital and vice versa.

Human capital – Estimation results

```
     Dependent variable: ln(MacroSolow[YL85])
      Number of observations: 75
 Variable      Coefficient  St. Error     t-statistic    Sign.
 Constant      7.8073650247  1.1905301242   6.5578895202  [0.0000]
ln((MacroSolow[N6085]/100)+0.05)
             -1.497194081   0.402574983   -3.7190440147  [0.0004]
ln(MacroSolow[IY]/100)
              0.7096271081  0.1503434765   4.7200392364  [0.0000]
ln(MacroSolow[SCHOOL]/100)
              0.7288214535  0.0950779272   7.6655168584  [0.0000]
    R^2adj. = 77.285811939%   DW = 2.3460
    R^2 = 78.206657401%       S.E. = 0.4539812636
    Residual sum of squares:  14.6330281236675
    Maximum loglikelihood: -45.1376297593491
    AIC =  1.3370034602        BIC =  1.4915026678
       F(3,71) = 84.92919 [0.0000]
    Normality: Chi^2(2) = 1.899269 [0.3869]
    Heteroskedasticity: Chi^2(1) = 0.132937 [0.7154]
    Functional form: Chi^2(1) = 1.665336 [0.1969]
    AR(1) in the error: Chi^2(1) = 2.420117 [0.1198]
    ARCH(1) in the error: Chi^2(1) = 1.295844 [0.2550]
```

Solow model – Estimation results

```
                        Ordinary least squares
     Dependent variable: ln(MacroSolow[YL85])
     Number of observations: 75
    Variable                 Coefficient  St. Error   t-statistic  Sign.
1 Constant                     5.3676983   1.540081    3.4853332   [0.0008]
2 ln((MacroSolow[N6085]/100)+0.05)
                              -2.0133899   0.5328300  -3.7786717   [0.0003]
3 ln(MacroSolow[IY]/100)
                               1.3253532   0.1706108   7.7682812   [0.0000]
  R^2adj. = 59.063938603%   DW = 1.9816
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  Residual sum of squares:  26.7434352866204
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  Functional form: Chi^2(1) = 0.456655 [0.4992]
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```

Results – Comments

- Improved fittness according to Solow model.
- Diagnostics OK (variables significant, heteroscedasticity OK, normality OK, DW-statistics is OK as well, Reset test gives good results, too).
- Signs – as expected.
- Not so high coefficients at investments.

Part 2: Convergence

Does the income of poor countries converge to the rich ones?

Convergence

- Rationale for convergence of income:
  - Diminishing returns in neoclassical models implie that countries with lower initial income will grow faster.
  - Therefore, poor and rich countries should converge in terms of income levels per capita.
- Is this convergence-hypothesis supported with the data?

Convergence

- Regress output growth over some period on a constant and initial income (Unconditional convergence)

$$\log\left(\frac{Y}{L}\right)_{i,t=2} - \log\left(\frac{Y}{L}\right)_{i,t=1} = a + b\,\log\left(\frac{Y}{L}\right)_{i,t=1} + \epsilon_i$$

- Problems => lack of data and also their reliability namely at t=1. (availability of data for poor countries before 1960)

Convergence - Results

Model 2: OLS estimates using the 75 observations 1-75
Dependent variable: LogGrowth

VARIABLE	COEFFICIENT	STDERROR	T STAT	P-VALUE
const	0.568026	0.432195	1.314	0.19287
l_YL60	-0.00197253	0.0547425	-0.036	0.97135

- Very poor results, negative convergence rate identified.
- Sample sensitive: years and selection of countries does matter, that's why in Romer's textbook slightly different results
- Why: poor growth in 80's in most developing countries etc.

Let's put it in a different way...

- Barro (1989):

In neoclassical growth models with diminishing returns, such as Solow (1956), Cass (1965) and Koopmans (1965), a country's per capita growth rate tends to be inversely related to its starting level of income per person. Therefore, in the absence of shocks, poor and rich countries would tend to converge in terms of levels of per capita income. However, this convergence hypothesis seems to be inconsistent with the cross-country evidence, which indicates that per capita growth rates are uncorrelated with the starting level of per capita product.

- Mankiw-Romer-Weil (1992):
  - Countries might have different steady states but if we control for the determinants of steady state (namely the saving rates), *conditional* convergence occurs.

Convergence and Solow model

- Conditional convergence:
  - Regress output growth on initial income and saving rates.

$$\log y(t) - \log y(0) = \beta_1 \log s_k + \beta_2 \log(n+g+\delta) + \beta_3 \log y_0$$

$$\log y(t) - \log y(0) = \beta_1 \log s_k + \beta_2 \log(n+g+\delta) + \beta_3 \log s_h + \beta_4 \log y_0$$

Conditional Convergence - Results

Model 3: OLS estimates using the 75 observations 1-75
Dependent variable: LogGrowth

VARIABLE	COEFFICIENT	STDERROR	T STAT	P-VALUE	
const	2.26937	0.847275	2.678	0.0092	***
logs	0.653203	0.103012	6.341	1.85e-08	***
logngd	-0.452568	0.304749	-1.485	0.1420	
l_YL60	-0.227296	0.0567518	-4.005	0.0002	***

- Much better results.
- Growth is negatively correlated with initial income if we controll for other variables.

Key points

1. Investigating the effect of various variables on growth: the cross-country regression framework
2. "Proxy" variable when considering un-measurable or unobserved variables in regression
3. Interpreting signs in regression – appropriate only for significant variables

Script

```
open /.../mrw.gdt
logs gdp85
genr s = log(inv/100)
genr ngd = log(popgrow/100+0.05)
smpl nonoil –dummy
smpl intermed --dummy
# model 1
ols l_gdp85 const s ngd
genr ls=log(school/100)
# model 2
ols l_gdp85 const s ngd ls
genr LogGrowth=l_gdp85-l_gdp60
# model 3
ols LogGrowth const l_gdp60
# model 4
ols LogGrowth const l_gdp60 s ngd
# model 5
ols LogGrowth const l_gdp60 s ngd ls
```

Note to problem sets

- Your solution should contain:
  - Few words about problem which you solve
  - Specification of model (=equation)
  - Method (usually OLS) and software used
  - Estimation results
  - Some diagnostics (test of homoscedasticity, normality and non-autocorrelated residuals, comments to significance of variables).
  - Interpretation of results.

Appendix

- Derivation of convergence rates
- TBA