- $(\mathbb{K}, +, \cdot)$ field

V set is a vector space over $\mathbb{K}$ with operations

$+ : V \times V \longrightarrow V$, $\cdot : \mathbb{K} \times V \longrightarrow V$

- $(V, +)$ is an abelian group
- $1 \cdot v = v \quad \forall v \in V$

  $\uparrow$ neutral element of $(\mathbb{K} \setminus 0, \cdot)$
- $a \cdot (b \cdot v) = (a \cdot b) \cdot v \quad \forall a, b \in \mathbb{K}, v \in V$
- $(a + b) \odot v = a \cdot v + b \cdot v$
- $a \cdot (v + w) = a \cdot v + a \cdot w$.

$\mathbb{R}^2$ as $\mathbb{R}$ vector space

$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x' \\ y' \end{pmatrix}$; $a \begin{pmatrix} x \\ y \end{pmatrix}$.

$a\left(\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x' \\ y' \end{pmatrix}\right)$

$= a \begin{pmatrix} x \\ y \end{pmatrix} + a \begin{pmatrix} x' \\ y' \end{pmatrix}$.

<u>very usefull</u>.

$\mathbb{K}^n = \left\{ \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} : a_i \in \mathbb{K} \right\}$

$+$ coordinate

$a \in \mathbb{K}$, $a \begin{pmatrix} a_1 \\ a_n \end{pmatrix} = \begin{pmatrix} a \cdot a_1 \\ a \cdot a_n \end{pmatrix}$

- V vector space over $\mathbb{K}$, $U \subseteq V$ subspace and

  i) $u, v \in U \Rightarrow u + v \in U$

  ii) $\forall u \in U, a \in \mathbb{K}, a \cdot u \in U$.

- $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$ that satisfy $x_1 + x_2 = 0$ this will be subspace of $\mathbb{R}^2$.

- $\{x \in \mathbb{R}^n : Ax = 0\}$ subspace of $\mathbb{R}^n$.

  $\shortparallel$

  Kernel of A

- $v_1, \dots, v_n \in V$, a linear combination is a sum $a_1 v_1 + \dots + a_n v_n$ for $a_i \in \mathbb{K}$.
- linear hull $\mathcal{L}(X) =$ all linear combinations of elements from X.

  $X \subseteq V$ subset.

- a set of vectors X is linearly independent if 0 cannot be expressed as a non trivial linear combination of vectors from X.

$\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ are li $\rightsquigarrow$ $a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ $\Big\}$ $a=0$ & $b=0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \begin{matrix}\| \\ \begin{pmatrix}a\\b\end{pmatrix}\end{matrix}$

- $X = \{v_1, \dots, v_n\}$ are li iff $\left(\sum a_i v_i = 0 \Rightarrow a_i = 0 \;\forall i\right)$.

- $\underline{\mathbb{R}^n}$: $X = \{v_1, \dots, v_m\} \in \mathbb{R}^n$ we want to see if $0 \in \mathcal{L}(X)$

$$\begin{pmatrix} — v_1 — \\ \\ \\ — v_m — \end{pmatrix} \xrightarrow{\text{row operations}} \begin{pmatrix} \text{(echelon form)} \\ 0 \cdots 0 \cdots 0 \end{pmatrix} \rightsquigarrow 0 \in \mathcal{L}(X).$$

- a nontrivial sol to $\underline{a_1} v_1 + \dots + \underline{a_m} v_m = 0$

$$\begin{array}{c} \begin{array}{cccc} a_1 & a_2 & & a_m \end{array} \\ \left(\begin{array}{cccc|c} | & | & & | & 0 \\ v_1 & v_2 & \cdots & v_m & \vdots \\ | & | & & | & 0 \end{array}\right) \end{array} \xrightarrow{\text{Gauss}} \begin{pmatrix} \text{(echelon form)} \end{pmatrix}$$

- $X = \{ v_1, v_2, v_3 \} \rightsquigarrow \mathcal{L}(X) \cap \mathcal{L}(Y) \setminus 0$

$Y = \{ w_1, w_2 \}$

$$a_1 v_1 + a_2 v_2 + a_3 v_3 = b_1 w_1 + b_2 w_2$$

$$a_1 \underline{v_1} + a_2 \underline{v_2} + a_3 \underline{v_3} + b_1 (-\underline{w_1}) + b_2 (\underline{-w_2}) = 0.$$

$$\left(\begin{array}{ccccc|c} | & | & | & | & | & 0 \\ v_1 & v_2 & v_3 & -w_1 & w_2 & \vdots \\ | & | & | & | & | & 0 \end{array} \right) \rightsquigarrow \text{solve.}$$