- App. of matrix mult:
  - Fibonacci numbers: 0, 1, 1, 2, 3, 5, ...

$F_0 = 0,\ F_1 = 1,\ \underline{F_2} = \underline{F_1} + \underline{F_0} = 1,\ \underline{F_3} = F_2 + F_1 = 2$

2 operations.

$$\boxed{F_{n+2} = F_n + F_{n+1}}$$

in general to compute F_{n+1} ~ n operations.

- $F_{n+2} = F_{n+1} + F_n \rightsquigarrow \begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix}}$

$$= \begin{pmatrix} F_{n+1} + F_n \\ F_{n+1} \end{pmatrix}$$

$$= \underset{M}{\underset{\shortparallel}{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}}}^{n+1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- $\log n$: $M^2, M^4, \ldots, M^{\log n}$

Inverse of a matrix:

- $I_n \in \mathbb{R}^{n\times n}$, for a matrix $A \in \mathbb{R}^{n\times n}$ its inverse (if exists) will be matrix $B \in \mathbb{R}^{n\times n}$ st $BA = I_n$.

$$\boxed{A^{-1} = \frac{A^*}{\det A}}$$

- $(A \mid I_n) \rightsquigarrow (I_n \mid B)$, $B = A^{-1}$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 7 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)_{I_3} \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -7 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & -7 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

row 2 = row 2 − 7 · row 3

row 1 = row 1 − 3 row 3

row 1 = row 1 − 2 row 2

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 11 \\ 0 & 1 & 0 & 0 & 1 & -7 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \rightsquigarrow A^{-1} = \begin{pmatrix} 1 & -2 & 11 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{pmatrix}$$

B

- Thm: $A \in \mathbb{R}^{n \times n}$ the following are eq.

  i) A is regular.

  ii) $\operatorname{rank} A = n$ $\leftarrow$!

  iii) $A \sim\sim I_n$

  iv) $Ax = 0$ has only the trivial solution.