op. with matrices.

$A \in \mathbb{R}^{n \times m}$ $\quad A = (a_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} = \begin{pmatrix} a_{11} & \vdots & a_{1m} \\ a_{n1} & & a_{nm} \end{pmatrix}$

$a_{ij} \in \mathbb{R}.$

- $+$: $A, B \in \mathbb{R}^{n \times m}$ $\quad (A+B)_{ij} = A_{ij} + B_{ij}$

$$\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}; \quad A + B \in \mathbb{R}^{n \times m}$$

- mult. by real number $\alpha \in \mathbb{R}$: $A \in \mathbb{R}^{n \times m} \Rightarrow \alpha A \in \mathbb{R}^{n \times m}$

$$(\alpha A)_{ij} = \alpha \cdot A_{ij} \;\Big|\; \begin{array}{l} \alpha = 2 \\ A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \end{array} \Rightarrow 2 \cdot \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 8 & 2 \end{pmatrix}.$$

- <u>TRANSPOSE</u>: $A \in \mathbb{R}^{n \times m} \longrightarrow A^T \in \mathbb{R}^{m \times n}$

$$(A^T)_{ij} = A_{ji}$$

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}.$$

$$(A^T)_{ii} = A_{ii}.$$

- product of two matrices: $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$

$$AB \in \mathbb{R}^{m \times p} \qquad (AB)_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 9 & 2 \cdot 1 + 3 \cdot 1 \\ & \end{pmatrix}.$$

- $$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{= I_3} \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{= A} = \begin{pmatrix} a_{11} & a_{12} & \\ & & \\ & & \end{pmatrix} = A.$$

- $I_3 A = A$; $A I_3 = A$.

- some exp with matrices = another expression.

$$(A + (B + C))_{ij} = ((A + B) + C)_{ij}$$