

# Systems of equations.

$$* \underline{Ax=b} \quad A \in \mathbb{R}^{m \times n}, \quad b \in \mathbb{R}^m, \quad x = (x_1, \dots, x_n)^T$$

$$A = \begin{pmatrix} a_{11} & a_{1n} \\ a_{m1} & a_{mn} \end{pmatrix}; \quad b = \begin{pmatrix} b_1 \\ b_m \end{pmatrix}$$

$$* \begin{array}{l} a_{11}x_1 + \dots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n = b_m \end{array} \quad \begin{array}{l} \cdot x \text{ is solution if } Ax=b. \\ \cdot \text{homogeneous } Ax=0. \end{array}$$

$$\cdot \underline{\text{line}}: \quad (a \ b) \begin{pmatrix} x \\ y \end{pmatrix} = c$$

$$\underline{\text{plane}}: \quad (a \ b \ c) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = d$$

• elementary equivalent row transformations

$(A|b) \sim (A'|b')$  if both have solutions.

i) mult by  $\lambda$   $\lambda \in \mathbb{R} \setminus 0$ .

ii) add row to another row

iii) add mult row to another row

iv) switch two rows

$$\begin{aligned} \cdot \quad x_1 + 2x_2 &= 5 \\ 2x_1 - x_2 &= 0 \end{aligned} \rightsquigarrow Ax = b$$

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}; \quad b = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$\left( \begin{array}{cc|c} 1 & 2 & 5 \\ 2 & -1 & 0 \end{array} \right) \sim \left( \begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -5 & -10 \end{array} \right) \sim \left( \begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 5 & 10 \end{array} \right)$$

$$\text{row 2} = \text{row 2} - 2 \text{ row 1}$$

$$\cdot \quad 5x_2 = 10 \Rightarrow x_2 = 2 \quad \Rightarrow \underline{\text{sol: } (1, 2)}$$

$$x_1 = 5 - 2x_2 = 5 - 4 = 1$$