- $\mathbb{R}^2 \ni \begin{pmatrix} x \\ y \end{pmatrix}$; $\mathbb{R}^3 \ni \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ | $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$

- How to describe a line in $\mathbb{R}^2$?

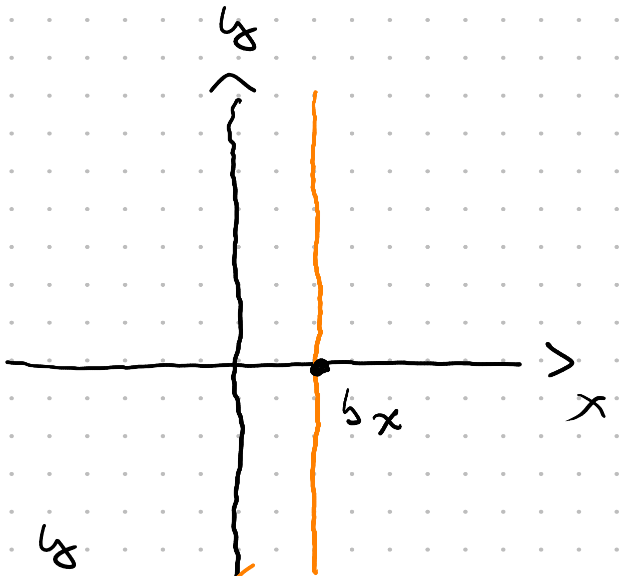
$$y = mx + c \rightsquigarrow \begin{pmatrix} x \\ mx+c \end{pmatrix}$$

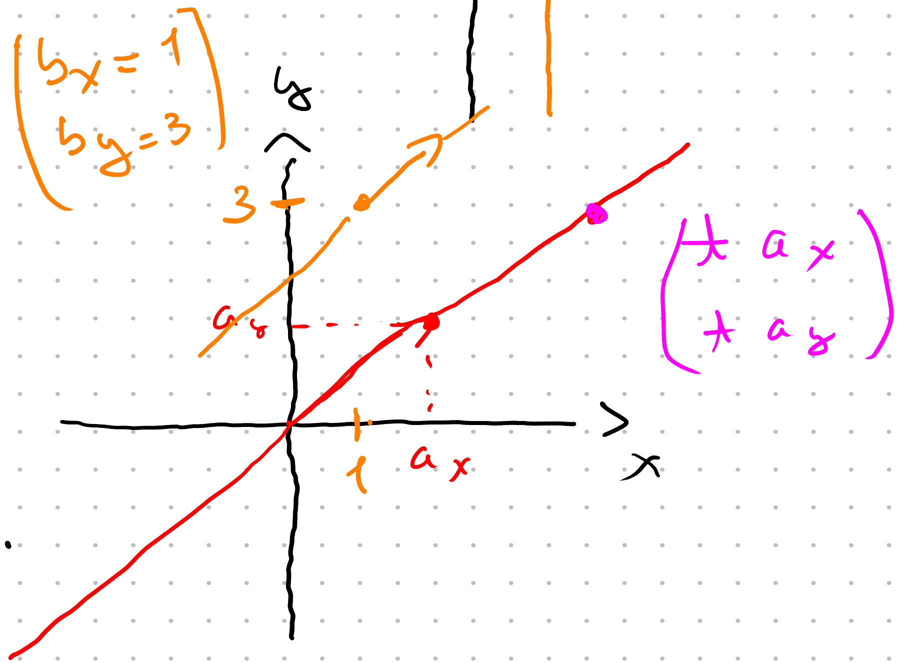
- parametric description

$$t \in \mathbb{R} ; \quad a = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$$

$$t \cdot \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} t \cdot a_x \\ t \cdot a_y \end{pmatrix} \in \mathbb{R}^2$$

$$t \cdot \begin{pmatrix} a_x \\ a_y \end{pmatrix} + \underbrace{\begin{pmatrix} b_x \\ b_y \end{pmatrix}}_{= 0} : t \in \mathbb{R}.$$





equation: $ax + by = c$

$a, b, c \in \mathbb{R}$; if all are non-zero : $y = -\frac{a}{b} x + \frac{c}{b}$

- $\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = ax + by$

$$\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = c$$

$$\boxed{\begin{pmatrix} a & b \end{pmatrix} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \right) = 0}$$

$$\|$$

$$\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \underbrace{\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}}_{= c} = 0$$

• How to describe a plane in $\mathbb{R}^3$?

<u>in $\mathbb{R}^2$</u>

$\uparrow (0,1)$

$\rightarrow (1,0)$

$\rightsquigarrow \quad (x,y) = x\cdot(1,0) + y\cdot(0,1).$

<u>in $\mathbb{R}^3$</u>: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$: $v = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \in \mathbb{R}^3$, $w = \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} \in \mathbb{R}^3$

• parametric description of a plane:

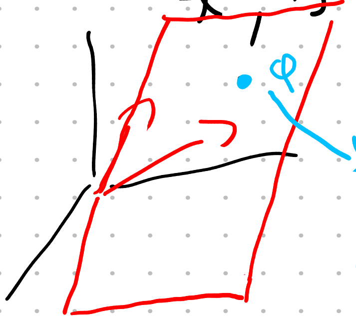
$t, s \in \mathbb{R}$; $[t\underline{v} + s\underline{w}] + p$

slopes or directions

$p = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$

for all $t, s \in \mathbb{R}$.

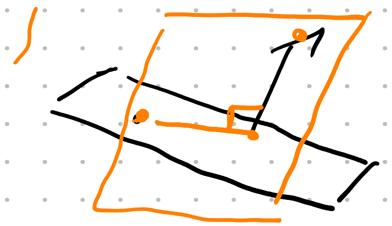
$$\begin{pmatrix} t v_x + s w_x \\ t v_y + s w_y \\ t v_z + s w_z \end{pmatrix}$$



$\to$ exist t, s st

$\varphi = *$

Equation description for the plane:



Line in $\mathbb{R}^2$:

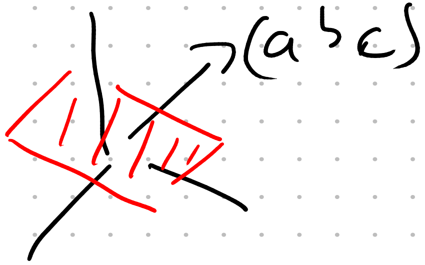
- $ax + by = c$

- Plane: points of the plane are the $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$ that satisfy.

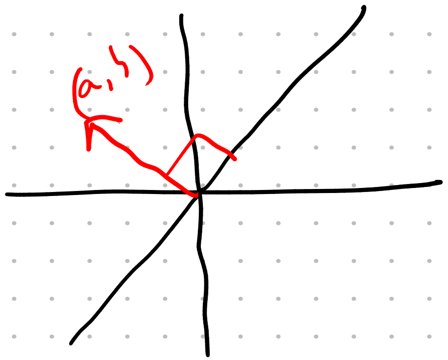
$$ax + by + cz = d$$

$$\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = d \rightsquigarrow \begin{pmatrix} a & b & c \end{pmatrix} \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \right) = 0$$

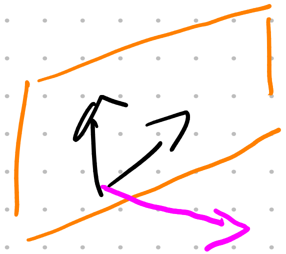
Interpretation: $d = 0$ $\quad (a, b, c) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$



line:



$(a\ \ b)\begin{pmatrix}x\\y\end{pmatrix}=0$



- parametric => equation for the plane:

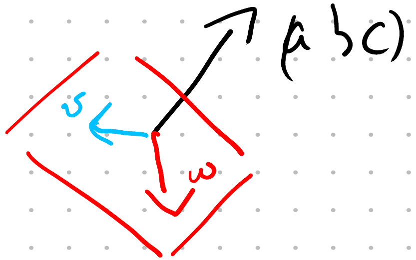
$$t\underline{v} + s\underline{w} + \underset{=0}{p} \rightsquigarrow (a\ \ b\ \ c)\begin{pmatrix}x\\y\\z\end{pmatrix} = d$$

- build (a,b,c) orthogonal to v and w:

$$v \times w : \begin{pmatrix} x & y & z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix} \Rightarrow \Big(\overset{x}{v_y w_z - v_z w_y},\ \overset{y}{-(v_x w_z - v_z w_x)},\ \overset{z}{v_x w_y - v_y w_x}\Big).$$

• equation $\Rightarrow$ parametric:

$$(a\ b\ c)\begin{pmatrix}x\\y\\z\end{pmatrix} = d \;\Rightarrow\; t\underline{v} + s\underline{w} + p.$$



(1 0 0)

1) pick $u'' \in \mathbb{R}^3$ not parallel to $\begin{pmatrix}a\\b\\c\end{pmatrix}$

$$v = \begin{pmatrix}a\\b\\c\end{pmatrix} \times u$$

$$w = v \times \begin{pmatrix}a\\b\\c\end{pmatrix}$$

if $a \neq 0$

$$ax + by + cz = d$$

$$x = \frac{1}{a}\big(d - b\underset{\overset{\shortparallel}{0}}{y} - c\underset{\overset{\shortparallel}{0}}{z}\big)$$

$$\rightsquigarrow\; \underline{t v} + \underline{s w} + p \qquad \begin{pmatrix}-d/a\\0\\0\end{pmatrix} = p.$$

$$v = (v_x, v_y, v_z)$$

$$\omega = (\omega_x, \omega_y, \omega_z)$$

$$v \times \omega : \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} v_x & v_y & v_z \\ \omega_x & \omega_y & \omega_z \end{pmatrix} \end{array}$$

$$v \times \omega = \left(v_y \omega_z - v_z \omega_y, -(v_x \omega_z - v_z \omega_x), v_x \omega_y - v_y \omega_x \right)$$