

Full name:

1	2	3	4	5	6

## 5th exam NMAI059 Probability and Statistics 1 – July 29, 2021

Write the number of the problem and your surname on each paper.

On this paper you may also write a pseudonym. In such case, your result will be announced with this pseudonym (otherwise, with your initials).

Do not write more than one problem on the same sheet of paper!

You have **150 minutes**.

No calculators, cell phones, ... are allowed during the exam. (Please mute your cell phones in advance.)

If the result contains expressions that are difficult to evaluate without a calculator, don't evaluate them ( $137 \times 173$  is as good, if not better, than 23701, you may leave  $\Phi^{-1}(0.975)$  unevaluated as well).

**Explain in detail** all calculations.

You may use one (handwritten) A4 cheat sheet.

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After the exam is marked everyone will be offered a grade of 1, ..., 5. You may improve this by one grade in an oral part, that is a 4 can be improved to a 3, a 3 to a 2, a 2 to a 1, but a 5 is a fail for this term of the exam.

Students writing the exam via Zoom must also attend the oral part (presumably also via Zoom) even if they don't desire improving the grade.

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**Explain in detail** all calculations.

1. (10 points)

In the table is the joint pmf of random variables  $X, Y$ . These random variables only take the values indicated in the table.

$x \backslash y$	1	2	3
1	1/4	1/6	1/12
2	1/6	1/4	1/12

- Determine  $P(X = 2 | Y = 1)$  and  $P(Y = 1 | X = 2)$ .
- Decide, whether the marginal distribution of  $X$  is uniform on  $\{1, 2\}$ .
- Decide, whether the marginal distribution of  $Y$  is uniform on  $\{1, 2, 3\}$ .
- Are  $X$  and  $Y$  independent?
- Compute  $\mathbb{E}(X + Y)$ .
- Compute  $\mathbb{E}(XY)$ .
- Compute  $\text{cov}(X, Y)$ .

2. (10 points) King Louis wants to have a male offspring to give him the name Louis. Every year his wife gives birth to exactly one child that is equally likely to be a boy and a girl, independently of the previous attempts. All children survive. When a boy is born, no more Louis's children will be born. We let  $S$  be the number of Louis's sons, and  $D$  the number of his daughters.

- Compute  $\mathbb{E}(S)$ .
- Compute  $\mathbb{E}(D)$ .

3. (10 points) Let  $X, Y \sim U(0, 1)$  be i.i.d. We define  $A = \min(X, Y)$  and  $B = \max(X, Y)$ .

- Determine the cumulative distribution function of  $B$ .
- Determine the probability distribution function of  $B$ .
- Compute  $\mathbb{E}(B)$ .
- Determine  $\mathbb{E}(X)$ ,  $\mathbb{E}(Y)$  and  $\mathbb{E}(A)$ .
- Determine  $\text{cov}(A, B)$ .

4. (10 points) (a) Define the notion joint probability distribution function of two random variables. How it be used to determine the marginal pdf of each of the coordinates?

(b) Define the notion of independence of several events. Can there be three events that are not independent, but every pair of them is?

5. (10 points) State the Central limit theorem. Explain what is it good for.

6. (10 points) State and prove Bayes' theorem (basic version for events).