3rd exam NMAI059 Probability and Statistics 1 – June 30, 2021

Write the number of the problem and your surname on each paper.
Do not write more than one problem on the same sheet of paper!
You have **150 minutes.**
No calculators, cell phones, ... are allowed during the exam. (Please silence your cell phones in advance.)
If the result contains expressions that are difficult to evaluate without a calculator, don't evaluate them (137×173 is as good, if not better, than 23701, you may leave $\Phi^{-1}(0.975)$ unevaluated as well).
Explain in detail all calculations.
You may use one (handwritten) A4 cheat sheet.

After the exam is marked everyone will be offered a grade of 1, ..., 5. You may improve this by one grade in an oral part, that is a 4 can be improved to a 3, a 3 to a 2, a 2 to a 1, but a 5 is a fail for this term of the exam.
Students writing the exam via Zoom must also attend the oral part (presumably also via Zoom) even if they don't desire improving the grade.

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1. (10 points) Decide if there are random variables X, Y on the same probability space such that
(a) $X \sim Pois(1/10)$, $Y \sim Bin(100, 1/10)$ and $P(X \leq Y) = 1$.
(b) $X \sim Pois(1/10)$, $Y \sim Bin(100, 1/10)$ and $P(X \geq Y) = 1$.
(c) $X \sim Bin(100, 1/2)$, $Y \sim Bin(100, 1/10)$ and $P(X \geq Y) = 1$.
(d) $X \sim Bin(100, 1/2)$, $Y \sim Bin(100, 1/10)$ and $P(X \leq Y) = 1$.

2. (10 points) Let $X \sim N(0, 1)$ and $Y = |X|$.
(a) What is the cumulative distribution function of Y?
(b) What is the probability density function of Y?
(c) Compute $\mathbb{E}(Y)$.
(d) Determine the median of Y.

3. (10 points) Peter has a (6-sided) dice, on which he rolls a six with probability p. Numbers 1, 2, ..., 5 all roll with the same probability.
(a) Determine the expected outcome of one roll of Peter's dice, in dependence on p.
(b) Find point estimate $\hat{p}$ by the moment method. What is your estimate if Peter rolled numbers 2, 6, and 3?
(c) Find point estimate $\hat{p}$ by the maximal likelihood method. Again provide an answer for rolled numbers 2, 6, and 3.

(The problems continue on the other side.)

4. (10 points) (a) Define the notion correlation of (two) random variables. What is the correlation $\varrho(X, X)$ for $X \sim U(0, 1)$?

(b) Define the notion independent random variables (continuous case, two variables). Give both equivalent formulations. Can we have independent X, Y such that $X \sim U(0, 1)$ and $Y \sim U(0, 1)$?

5. (10 points) Explain how to do the goodness of fit test.

6. (10 points) State and prove the theorem about total expectation. Variant for discrete random variables is sufficient.