SAMPLE exam NMAI059 Probability and Statistics 1 – 2020/21

Write the number of the problem and your surname on each paper. Do not write more than one problem on the same sheet of paper!

1. (10 points) The following table describes the joint probability function $p_{X,Y}(x, y)$ of the random vector (X, Y). These random variables only take values indicated in the table. We know that $\mathbb{E}(X) = 1/2$.

x \ y	-1	0	1
0	a	$1/8$	$1/4$
1	$1/8$	b	$1/8$

(a) Determine a and b.
(b) Decide whether X and Y are independent.

2. (10 points) Peter repeatedly tries to defeat a stronger opponent in chess. If Peter wins, he gains 10 points, otherwise (even in the event of a draw) he loses 2 points. Peter wins with probability $1/4$.

(a) In how many rounds does Peter win for the first time (on average)?

(b) If Peter has 6 points at the beginning, with what probability will he be at zero after at most five rounds?

(c) What is the distribution of S, Peter's score after five games, if he has zero points at the start (negative points are allowed)? Describe the probability mass function of S.

(d) What event has higher probability: "Peter will have at least 20 points after ten rounds" or "Peter will have at least 200 points after one hundred rounds"?

3. (10 points) You're throwing a party for 100 guests and wondering how many sandwiches to order. You know from experience that the number of sandwiches eaten by a random guest follows a Poisson distribution with a mean of 3. Approximately how many sandwiches do you need to order so that with probability 0.95 no guest will go hungry?

(Use an appropriate limit theorem.)

(The problems continue on the other side.)

4. (10 points) (a) Define the concept of probability density function of a random variable X.

(b) Describe how to use it to determine $\mathrm{var}(X)$.

5. (10 points) State the theorem – the universality of uniform distribution. Explain how it can be used.

6. (10 points) State the theorem – the weak law of large numbers. Prove it.

You have **150 minutes.**

No calculators, cell phones, ... are allowed during the exam. (Please silence your cell phones in advance.)

If the result contains expressions that are difficult to evaluate without a calculator, don't evaluate them (137×173 is as good, if not better, than 23701, you may leave $\Phi^{-1}(0.975)$ unevaluated as well).

Explain in detail all calculations. You may use one (handwritten) A4 cheat sheet.