8. Determine the pair-correlation function of a binomial point process, provided it exists.

$$\Phi \sim Bi(n, \nu, B) \qquad B \in \mathcal{B} \qquad \Phi(B_1) \sim Bi\left(n, \frac{\nu(B_1)}{\nu(B)}\right)$$

- $\lambda(x) = ?$ $\qquad \Lambda(B_1) \overset{?}{=} \int_{B_1} \lambda(x)dx \qquad B_1 \subseteq B, \ B_1 \in \mathcal{B}$

$\Lambda(B_1) = \mathbb{E}\Phi(B_1)$

assume:

$$\underbrace{\nu(B_1) = \int_{B_1} h(x)dx}$$

$$\mathbb{E}\Phi(B_1) = \underbrace{n \cdot \frac{\nu(B_1)}{\nu(B)}} = \int_{B_1} \frac{n\, h(x)}{\nu(B)} dx \quad \Rightarrow \lambda(x) = \frac{n\, h(x)}{\nu(B)}$$

- $\lambda^{(2)}(x,y) = ?$ $\qquad \alpha^{(2)}(B_1 \times B_2) = \int_{B_1}\int_{B_2} \lambda^{(2)}(x,y)\,dx\,dy$

B_1, B_2 disjoint $\subseteq B$

$$\alpha^{(2)}(B_1 \times B_2) = \mathbb{E} \sum_{X_1, X_2 \in \operatorname{supp}\Phi}^{\neq} \mathbb{1}(X_1 \in B_1)\mathbb{1}(X_2 \in B_2) \overset{4.7}{=} n(n-1)\frac{\nu(B_1)}{\nu(B)} \cdot \frac{\nu(B_2)}{\nu(B)}$$

$\nu(B_1) = \int_{B_1} h(x)dx \qquad \nu(B_2) = \int_{B_2} h$

$$\Rightarrow \lambda^{(2)}(x,y) = n(n-1)\, h(x)h(y) / \nu(B)^2, \qquad x, y \in \mathbb{R}^d$$

- $$g(x,y) = \frac{\lambda^{(2)}(x,y)}{\lambda(x)\lambda(y)} = \frac{n-1}{n}, \quad \text{provided } h(x)h(y) > 0$$

dx $\quad dy$

$x \quad y \qquad \lambda^{(2)}(x,y)\,|dx|\,|dy| \approx \mathbb{P}\big(\underbrace{\Phi(dx) \geq 1}, \underbrace{\Phi(dy) \geq 1}\big)$

dependent events due to fixed number of points