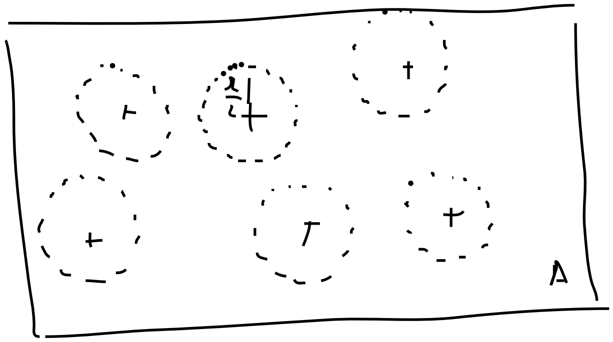
9. For a point process with the hard-core distance $r > 0$ and the intensity λ we define the *coverage density* as $\tau = \lambda|b(o, r/2)|$. It is in fact the mean volume fraction of the union of balls with the centers in the points of the process and the radii $r/2$. Determine the maximum possible value of τ for the following models:

a) Matérn hard-core process type I,

b) Matérn hard-core process type II.



$|A| = 1$

$\tau = E|\bigcup b$ ...

$x_i \in \text{sup}$ ...

$$\tau_I = \lambda_I \cdot |b(o, \tfrac{r}{2})|$$

a) type I: $\lambda_I = \lambda_p e^{-\lambda_p \omega_d r^d}$

λ_p ... intensity o... original Poi...

b) type II: $\lambda_{II} = \dfrac{1 - e^{-\lambda_p \omega_d r^d}}{\omega_d r^d}$

b) $\tau_{II}^{(\lambda_p, r)} = \lambda_{II} \cdot \omega_d \left(\frac{r}{2}\right)^d = \frac{1}{2^d}\left(1 - e^{-\lambda_p \omega_d r^d}\right)$

argmax $\tau_{II}(\lambda_p, r)$? ✗ $\quad \tau_{II}(\lambda_p, r^d) < \tau_{II}(\lambda_p + 1, r^d)$

but: $\sup\limits_{\lambda_p > 0, r > 0} \tau_{II}(\lambda_p, r) = \frac{1}{2^d}$ ... in plane: $\sup \tau_{II} =$ ...

a) $\tau_I(\lambda_p, r) = \lambda_I \cdot \omega_d \left(\frac{r}{2}\right)^d = \lambda_p e^{-\lambda_p \omega_d r^d} \cdot \omega_d r^d \left(\frac{1}{2}\right)^d$

$x = \lambda_p \omega_d r^d \qquad f(x) = c\, x \cdot e^{-x}, \; x > 0$

$f'(x) = c\left(e^{-x} - x e^{-x}\right) = c e^{-x}(1 - x)$

$f'(x) = 0 \iff x = 1 \quad \ldots \; f''(x) = ?, \; f''(1) < 0 \; ?$

$x = 1 \iff \lambda_p \omega_d r^d = 1$ .... any combination of (λ_p, r) sa... that $\lambda_p \omega_d r^d = 1$ maximizes τ_I

$\sup \tau_I(\lambda_p, r) = \left(\frac{1}{2}\right)^d e^{-1}$ ... in plane $(d = 2) \Rightarrow \approx 0{,}092$