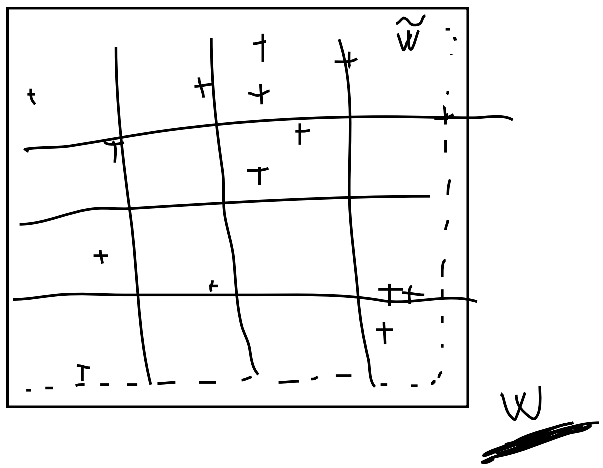
5. Consider the point pattern $\{x_1, \dots, x_n\}$ observed in a compact observation window $W \subset \mathbb{R}^2$ and assume it is a realization of a stationary point process. How to estimate its intensity? How to estimate the values $F(r)$ and $G(r), r > 0$?

$\lambda(u) \equiv \lambda =$ „expected # points in a set of unit area"



$\Lambda(B) = \mathbb{E}\,\Phi(B) = \lambda \cdot |B|$

disjoint

$W_1, \dots, W_M \qquad |W_i| = 1$

$\Phi(W_1), \dots, \Phi(W_M)$

$$\frac{1}{M}\sum_{i=1}^{M} \Phi(W_i) = \frac{1}{M}\,\Phi\Big(\bigcup_{i=1}^{M} W_i\Big) =$$

$$\underbrace{\frac{1}{|W|}\cdot \Phi(W) = \hat{\lambda}} \Rightarrow \quad = \frac{1}{\left|\bigcup_{i=1}^{M} W_i\right|}\,\Phi\Big(\underbrace{\bigcup_{i=1}^{M} W_i}_{\tilde{W}}\Big) = \frac{1}{|\tilde{W}|}\cdot\Phi(\tilde{W})$$

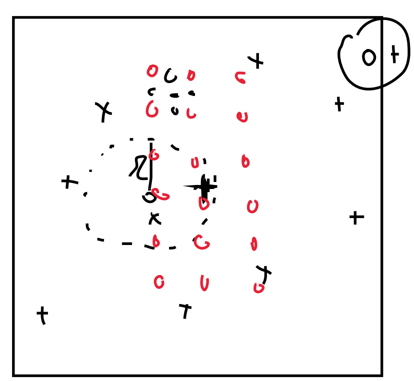
$\lambda =$ „expected # of points per unit area"

$$\mathbb{E}\,\hat{\lambda} = \frac{1}{|W|}\cdot \mathbb{E}\,\Phi(W) = \frac{1}{|W|}\,\Lambda(W) = \frac{1}{|W|}\,\lambda|W| = \lambda \quad \Rightarrow \text{unbiased estimator}$$

$$\left[\ \widehat{\Lambda(B)} = \Phi(B)\ \right]$$

$$G(r) = P^{!}_{o}\big(\{\varphi \in \mathcal{N} : \varphi(b(o,r)) > 0\}\big), \quad r \geq 0, \quad \text{stationarity!}$$

$\{x_1, \dots, x_n\} \subset W$

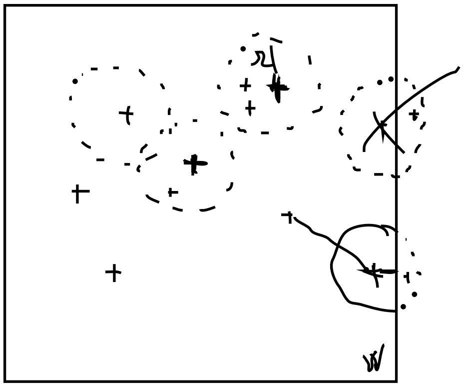


$r > 0$: $\quad F(r) = P(D_o \leq r)$

$I = \{a_1, \dots, a_N\} \quad D_x \ \dots$ stationarity

$$\widehat{F(r)} = \frac{1}{N}\sum_{i=1}^{N} \mathbb{1}\Big(\min_{j=1,\dots,n} d(a_i, x_j) \leq r\Big)$$

assuming $\bigcup_{i=1}^{N} b(a_i, r) \subset W$



border correction, minus sampli

$$\frac{1}{n}\sum_{i=1}^{n} \mathbb{1}\Big(b(x_i, r) \cap \{x_1, \dots, x_{i-1}, x_{i+1}, \dots$$

$$(1 - \mathbb{1}(\dots > r) \cdot w_i)$$

$$\widehat{G}(r) = \frac{1}{n} \sum \mathbb{1}\Big(\min_{j=1,\dots,i-1,i+1,\dots,n} d(x_i, x_j) \le r \Big)$$

↳ negatively biased (edge effects)

$$\widehat{G}(r) = \frac{1}{n(r,W)} \sum_{i=1}^{n} \mathbb{1}(\dots \le r) \cdot \mathbb{1}\Big(b(x_i, r)$$

$$n(r, W) = \sum_{i=1}^{n} \mathbb{1}(b(x_i, r) \subset W)$$

→ problems with monotonicity : $G(r)$ is non-decrea

$\widehat{G}(r)$ may decrease with $r \nearrow$ due to ignoring more points

(Kaplan-Meier estimators deal with censoring

edge effect