12.4: $Y_i = \begin{cases} 1 & \text{... i-tý bažant přežije} \\ 0 & \text{... jinak} \end{cases}$ $\qquad i = 1, \dots, 10$

$X = \sum_{i=1}^{10} Y_i$ = počet přeživších bažantů

$$\mathbb{E}X = \sum_{i=1}^{10} \mathbb{E}Y_i = \sum_{i=1}^{10} \left(1 \cdot \mathbb{P}(Y_i = 1) + 0 \cdot \mathbb{P}(Y_i = 0) \right) = 10 \left(1 - \frac{p}{10}\right)^{10}$$

$$\mathbb{P}(Y_i = 1) = \sum_{k=0}^{10} \mathbb{P}(Y_i = 1 \mid \text{vybrán } k\times) \cdot \mathbb{P}(\text{vybrán } k\times) = \sum_{k=0}^{10} (1-p)^k \binom{10}{k} \left(\dots\right.$$

$= \dots (*)$

$k = 0, \dots, 10$: $\mathbb{P}(Y_i = 1 \mid \text{vybrán } k\times) = (1-p)^k$

$$\left[\mathbb{P}(X_i = k) = \right] \mathbb{P}(\text{vybrán } k\times) = \binom{10}{k} \left(\frac{1}{10}\right)^k \left(1 - \frac{1}{10}\right)^{10-k}$$ ... binomické rozděle

$1 \cdot 1 \cdot (9/10)^{10}$

$\Rightarrow \mathbb{P}(Y_i = 1 \mid \text{vybrán } k\times) \cdot \mathbb{P}(\text{vybrán } k\times) = \mathbb{P}(Y_i = 1, \text{vybrán } k\dots$

$$(*) = \left(\frac{1}{10}(1-p) + \frac{9}{10} \right)^{10} = \left(1 - \frac{p}{10}\right)^{10}$$

$\frac{p}{10} = \mathbb{P}($ j-tý lovec $\underbrace{\text{vybral}}_{\frac{1}{10}}$ i-tého bažanta, $\underbrace{\text{tref}}_{p}\dots$

$1 - \frac{p}{10} = \mathbb{P}($ j-tý lovec nezabil i-tého bažan$\dots$

$p = \frac{1}{2} \Rightarrow \mathbb{E}X \doteq 5{,}987$, $\qquad p = 0 \Rightarrow \mathbb{E}X = 10$

$p = 1 \Rightarrow \mathbb{E}X \doteq 3{,}487$