$\nearrow$ in $\mathbb{R}^d$

4. Show that for a homogeneous Poisson process with the intensity λ it holds that PI $=$ CE $= 1$, $F(r) = G(r) = 1 - \mathrm{e}^{-\lambda\omega_d r^d}$ and $J(r) = 1$.

$D_o = \inf\{r \geq 0 : \Phi(b(o,r)) > 0\}$ = „smallest distance from o to a point of the process"

$PI = \lambda \cdot \omega_d \cdot \mathbb{E}(D_o)^d$

$CE = \frac{d(\lambda\omega_d)^{1/d}}{\Gamma(1/d)} \mathbb{E}_o^! D_o$

$F(r) = \mathbb{P}(D_o \leq r) = \mathbb{P}(\Phi(b(o,r)) >$

$G(r) = \ldots$

$J(r) = \frac{1-G(r)}{1-F(r)}, \ r > 0 : F(r) <$

$F(r) = \mathbb{P}(\Phi(b(o,r)) > 0) = 1 - \mathbb{P}(\underbrace{\Phi(b(o,r))}_{\hookrightarrow Po(\lambda\omega_d r^d)} = 0) = 1 - e^{-\lambda \cdot \omega_d \cdot r^d}, \ r >$

$= \mathbb{P}(D_o \leq r)$ ... dist. f. of D_o

p.d.f. of D_o : $f(r) = e^{-\lambda\omega_d r^d} \cdot \lambda\omega_d d r^{d-1}, \ r > 0$.

PI: $\mathbb{E}(D_o)^d = \int_0^\infty r^d f(r)\,dr = \int_0^\infty r^d \underbrace{r^{d-1}\lambda\omega_d d\, e^{-\lambda\omega_d r^d}\, dr}_{dx}$

$= \left| \begin{array}{l} x = \lambda\omega_d r^d \implies r^d = \frac{x}{\lambda\omega_d} \\ dx = \lambda\omega_d d r^{d-1} dr \end{array} \right.$

$= \int_0^\infty e^{-x}\frac{x}{\lambda\omega_d}dx = \frac{1}{\lambda\omega_d}\underbrace{\int_0^\infty x e^{-x} dx}_{\Gamma(2) = (2-1)! = 1} = \frac{1}{\lambda\omega_d}\cdot 1 = \frac{1}{\lambda\omega_d}$

$\implies PI = \lambda\omega_d \cdot \frac{1}{\lambda\omega_d} = 1$ for homogeneous Poisson p.p.

CE: $\mathbb{E}_o^! D_o = \int \underset{\downarrow\, d(o,\varphi)}{D_o}\, P_o^!(d\varphi) \overset{\text{Slivnyak}}{=} \int D_o\, \underset{\uparrow\,\text{distribution of } \Phi}{\Pi}(d\varphi) = \mathbb{E} D_o$

$D_o \geq 0$ a.s. $\implies \mathbb{E}D_o = \int_0^\infty (1 - F(r))\,dr = \int_0^\infty e^{-\lambda\omega_d r^d} dr = \ldots$

$$\ldots = \left|\begin{array}{l} x = \lambda \omega_d r^d \\ dx = \lambda \omega_d d\, r^{d-1}\, dr \end{array}\right. \qquad dr = \frac{dx}{\lambda \omega_d d\, r^{d-1}} = \frac{dx}{\lambda \omega_d d} \cdot \Big($$

$$r^{d-1} = \left(\frac{x}{\lambda \omega_d}\right)^{\frac{d-1}{d}}$$

$$\ldots = \int_0^\infty e^{-x} \frac{1}{\lambda \omega_d d} \cdot (\lambda \omega_d)^{\frac{d-1}{d}} \cdot x^{\frac{1}{d}-1}\, dx =$$

$$= \frac{1}{d} (\lambda \omega_d)^{\cancel{1} - \frac{1}{d} \cancel{-1}} \cdot \underbrace{\int_0^\infty x^{\frac{1}{d}-1} e^{-x}\, dx}_{= \Gamma\left(\frac{1}{d}\right)} = \frac{1}{d} \cdot \frac{1}{(\lambda \omega_d)^{\frac{1}{d}}}$$

$$\Rightarrow CE = 1 .$$

$$G(r) = P_o^!\big(\{\varphi \in N : \varphi(b(o,r)) > 0\}\big) =$$

Slivnyak

$$= \Pi\big(\{\varphi \in N : \varphi(b(o,r)) > 0\}\big) = \mathbb{P}\big(\Phi(b(o,r)) > 0\big) = F(r)$$

$$= 1 - e^{-\lambda \omega_d r^d}, \quad r \geq 0 .$$

$$G(r) = F(r) \Rightarrow J(r) = \frac{1 - G(r)}{1 - F(r)} = 1, \quad r > 0 .$$