



3. Consider independent random variables U_1 and U_2 with uniform distribution on the interval $[0, a]$, $a > 0$, and the point process Φ in \mathbb{R}^2 defined as

$$\Phi = \sum_{m, n \in \mathbb{Z}} \delta_{(U_1 + ma, U_2 + na)}$$

Determine the Palm distribution and the reduced second-order moment measure of the process. Express its contact distribution function and the nearest-neighbour distribution function.

$\omega_\lambda = \gamma$

Palm distribution P_x : degenerate ... $\Phi = \Phi^x = \sum_{m_1, m_2 \in \mathbb{Z}} \delta_{(x_1 + m_1, x_2 + m_2)}$

$$P_x^! : \Phi = \Phi^x = \sum_{(m_1, m_2) \in \mathbb{Z}^2 - \{(0,0)\}} \delta_{(x_1 + m_1, x_2 + m_2)} = \mathcal{C}_1^x - \delta_{(x_1, x_2)}$$

Reduced 2nd order moment measure:

$$\lambda \mathcal{K}(B) = \mathbb{E}_0^! \Phi(B), \quad B \in \mathcal{B}(\mathbb{R}^2)$$

↑ stationary point processes (Remark 37)

$$= \mathcal{C}_2^0(B) = \sum_{(m_1, m_2) \in \mathbb{Z}^2 - \{(0,0)\}} \mathbb{1}_{((ma, ma) \in B)}$$

$$\lambda = \frac{1}{a^2} \dots \text{from earlier exercise} \rightarrow \mathcal{K}(B) = a^2 \cdot \sum_{(m_1, m_2) \in \mathbb{Z}^2} \dots$$

Nearest neighbour distance distribution function:

$$r > 0 \quad G(r) = P_0^! \left(\{ \gamma \in \mathcal{N} : r(\gamma, r) > 0 \} \right)$$

$$G(r) = \mathbb{1}_{(a \leq r)}$$



$$F(r) = P(\Phi(B(r, r)) > 0)$$

