... diffuse $\Rightarrow$ simple point proc

$\Lambda(B) = \lambda \cdot |B|$, $B \in \mathcal{B}(\mathbb{R}^d)$ $\Downarrow$

$\lambda > 0$ dist. determined void pr

1. Show that a homogeneous Poisson point process is stationary and isotropic. Is there any stationary non-isotropic Poisson point process?

on $\mathbb{R}^d$

[if $\lambda = 0 \Rightarrow \mathbb{E}\,\Phi(B) = \Lambda(B) = 0$ & $\Phi(B) \geq 0 \Rightarrow \Phi(B) = 0$

stationarity $\quad \underbrace{\Phi \overset{d}{=} t_y \Phi}_{\text{„measure"}} = \underbrace{\Phi + y}_{\text{„support"}} \quad \forall y \in \mathbb{R}^d$

$$(t_y \Phi)(B) = \Phi(B - y)$$

$\Phi(B) \sim Po(\underbrace{\Lambda(B)}_{\lambda|B|})$ ... invariant w.r.t. shifts

$$P((t_y\Phi)(B) = 0) = P(\Phi(B-y) = 0) \underset{= e^{-\Lambda(B-y)}}{=} e^{-\lambda|B-y|} = e^{-\lambda|B|} = P(\Phi(B) = 0) \quad \forall B \in \mathcal{B}_0(\mathbb{R}^d)$$

$\Rightarrow \Phi \overset{d}{=} t_y \Phi \quad \forall y \in \mathbb{R}^d \Rightarrow$ stationarity

isotropy: R_σ .. rotation operator around the origin

$\Phi \overset{d}{=} R_\sigma \Phi \quad \forall R_\sigma$? $\qquad (R_\sigma \Phi)(B) = \Phi(R_\sigma^{-1} B)$

repeat the same argument as above with rotations

Leb. measure invariant w.r.t. all rigid motions

extra question: distribution of $\Phi \sim Po(\Lambda)$ determined by intensity measure $\Rightarrow$ Λ translation inv. $\to$ stationarity

rotation inv. $\to$ isotropy

Looking for measure Λ $\underbrace{\text{translation inv.}}$ but not rotation in

only multiples of Leb. measure

stationarity $\Rightarrow \Lambda(B) = \lambda|B| \Rightarrow$

$\Rightarrow$ homogeneity $\Rightarrow$ isotropy $\Rightarrow$ NO stationary Poisson processes which are not isotropic

[$\lambda(u) = e^{-\|u\|}$, $u \in \mathbb{R}^d$]