NMFM402 – Mathematics of Non-Life Insurance 2

GLM in loss reserving

Practical 8

Recall the use of GLM for setting the loss reserves and approximation of the mean square error of prediction (MSEP) of future claims from [1] or [2].

Exercise 1: Think of the possibilities to determine prediction intervals for future claims and for loss reserves (sum of future claims). Why is this important?

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Solution: If the reserves were set at the level of predicted future claims by GLM (the so called "best estimate"), the probability that the reserve would be insufficient (true claims would exceed the reserve) would be approx. 50 %, which is unacceptable. Therefore, we need to add a "safety margin"to loss reserves – a buffer that would absorb potential fluctuations of future claims (caused either by error in our estimates or by the process variance). To determine the size of the margin, we are interested in distribution or quantiles of the future claim, given the data that we have.

In general, determining prediction intervals in GLM methodology is complicated and not straightforward (in contrast to classical linear models). We can consider various approximations:

1. Assuming our prediction by GLM are unbiased point estimates of expected future claims (this is true at least asymptotically) and considering the mean square error of prediction (MSEP, see [1], section 2.4.3) as an approximation of variance of total future claims, we can construct the prediction intervals by combining these with quantiles of appropriate distribution. A natural (but not the only one) choice is the normal distribution, this choice can be reasoned by application of the central limit theorem to the sum of independent future claims.

2. Another option is based on the so called bootstrap technique – a popular and computationally demanding technique to reconstruct distribution of a certain statistics (function of data) by resampling from empirical distribution of data (obtain from i.i.d. observations). In the context of loss reserving via GLM, we can apply the parametric bootstrap do the following steps:

   B1 Estimate parameters of GLM using MLE on data in the triangle.

   B2 Estimate expectations of historical claims (i.e. fit the data in the triangle by GLM) – denote by
   $$\hat{x_{ij}} = \hat{\mu}_i \hat{\gamma}_j, \quad i + j \leq I$$

   B3 Calculate (or rather estimate) the Pearson's residuals (these are considered to be i.i.d.):
   $$Z_{ij} = \frac{X_{ij} - \hat{x_{ij}}}{\sqrt{V(\hat{x_{ij}})}}, \quad i + j \leq I$$
   where V is the variance function in the chosen EDF distribution.

   B4 The residuals $\{Z_{ij} : i + j \leq I\}$ represent the empirical distribution. Sample i.i.d. bootstrap residuals from this distribution:
   $$Z^*_{ij} \sim \{Z_{ij} : i + j \leq I\} \quad \text{(sampling with replacement)}$$

B5 Using bootstrap residuals, construct new (artificial) triangle

$$X_{ij}^{*} = \hat{x_{ij}} + \sqrt{V(\hat{x_{ij}})} Z_{ij}^{*}, \quad i + j \leq I.$$

B6 Use the new (artificial) triangle to fit new GLM model (fit new parameters) and calculate corresponding future claims (bootstrap loss reserve).

B7 Repeat steps B4 - B6 many times to obtain sufficient number of realizations of bootstrap loss reserves. The empirical distribution of these artificial reserves (future claims) is an approximation of true distribution of future claims given our data. Take appropriate quantiles of the empirical bootstrap distribution as an approximation of prediction intervals. Other characteristics (such as MSEP) can be calculated from the empirical distribution as well.

For more details, see [2], Chapter 7 (esp. Chapter 7.3)

3. If you have any other idea, how to approach this problem, or find something else in the literature, we can discuss during practicals.

Reference

[1] L. Mazurová *Mathematics of Non-life Insurance 2 - lecture notes.*Version March 2021. Available online at Moodle: https://dl1.cuni.cz/pluginfile.php/1162656/mod_resource/content/2/MNP2LectureNotes.pdf

[2] M. V. Wüthrich, M. Merz: *Stochastic Claims Reserving Methods in Insurance*, The Wiley Finance Series, John Wiley & Sons, Chichester, 2008