8. Dispersion of a random variable $\Phi(B)$ is defined as

$$D(\Phi(B)) = \frac{\operatorname{var}\Phi(B)}{\mathsf{E}\,\Phi(B)}, \quad B \in \mathcal{B}_0.$$

Show that

a) for a Poisson process $D(\Phi(B)) = 1$,

b) a binomial process is underdispersed, i.e. $D(\Phi(B)) \le 1$,

c) a Cox process is overdispersed, i.e. $D(\Phi(B)) \ge 1$.

Homework no. 3:

- for using Campbell Th… the process Φ needs to be simple (assume, or p.c.f of Φ exists)
- $\Phi_p(B) = \Phi(B \times W_2)$

$\Lambda_p(B) = \mathbb{E}\,\Phi_p(B) = \ldots$

$\ldots = \Lambda(B \times W_2)$

a) $\Phi(B) \sim Po(\Lambda(B)) \Rightarrow \mathbb{E}\,\Phi(B) = \operatorname{var}\Phi(B) = \Lambda(B)$

$\Rightarrow D(\Phi(B)) = 1$

b) $\Phi \sim B_{0,m,\nu}$, $B \subseteq B_0$ .... $\mathbb{E}\,\Phi(B) = m\frac{\nu(B)}{\nu(B_0)}$, $\operatorname{var}\Phi(B) = m\frac{\nu(B)}{\nu(B_0)}\Big(1$

$$D(\Phi(B)) = \frac{\operatorname{var}\Phi(B)}{\mathbb{E}\,\Phi(B)} = 1 - \frac{\nu(B)}{\nu(B_0)} \le 1$$

c) $\Phi \sim$ Cox process with random driving measure $\angle$ ... „Lámed" (Phoenician

... conditionaly on realization of $\angle$, Φ is Poisson proces… with intensity measure $\angle$.

$$\mathbb{E}\,\Phi(B) = \mathbb{E}\big[\mathbb{E}[\underbrace{\Phi(B)\,|\,\angle}_{\sim Po(\angle(B))}]\big] = \mathbb{E}\big[\angle(B)\big] = \mathbb{E}\angle(B) = \Lambda($$

$$\mathbb{E}\,\Phi(B)^2 = \mathbb{E}\big[\mathbb{E}[\Phi(B)^2\,|\,\angle]\big] = \mathbb{E}\big[\angle(B) + \angle(B)^2\big]$$

$$\operatorname{var}\Phi(B) = \mathbb{E}\angle(B) + \underbrace{\mathbb{E}\angle(B)^2 - (\mathbb{E}\angle(B))^2}_{} =$$

$$= \mathbb{E}\angle(B) + \operatorname{var}\angle(B)$$

$$D(\Phi(B)) = \frac{\operatorname{var}\Phi(B)}{\mathbb{E}\,\Phi(B)} = 1 + \underbrace{\frac{\operatorname{var}\angle(B)}{\mathbb{E}\angle(B)}}_{\ge 0} \ge 1$$

... assume $\mathbb{E}\angle(B) > 0$ $\big(= \Lambda(B)\big)$