$$f_Y(x) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}, \quad x>0$$

$$a, b > 0$$

$$L(B) = Y \cdot \Lambda(B)$$

↳ det

11. Let Y be a random variable with a gamma distribution. Show that the corresponding mixed Poisson process Φ is a negative binomial process, i.e. that $\Phi(B)$ has a negative binomial distribution for every $B \in \mathcal{B}_0$.

assume: $a \in \mathbb{N}$ ... $\Gamma(a) = (a-1)!$

choose $k = 0, 1, 2, \ldots$

$$\mathbb{P}(\Phi(B) = k) = \mathbb{E}\, \mathbb{1}(\Phi(B) = k) = \mathbb{E}\Big[\mathbb{E}\big[\underbrace{\mathbb{1}(\Phi(B) = k)}_{Po(Y\cdot\Lambda(B))} \,\big|\, Y\big]\Big] =$$

$$= \mathbb{E}\left[e^{-Y\Lambda(B)} \cdot \frac{(Y\Lambda(B))^k}{k!}\right] =$$

$$= \int_0^\infty e^{-x\Lambda(B)} \cdot \frac{x^k \Lambda(B)^k}{k!} \cdot \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}\, dx =$$

$$= \left[\int_0^\infty e^{-x(b+\Lambda(B))} x^{k+a-1} \cdot \frac{(b+\Lambda(B))^{k+a}}{\Gamma(k+a)}\, dx\right] \frac{\Gamma(k+a)}{(b+\Lambda(B))^{k+a}} \frac{\Lambda(B)}{k!}$$

$$= [1] \cdot \frac{b^a}{(b+\Lambda(B))^{k+a}} \cdot \Lambda(B)^k \cdot \frac{(a+k-1)!}{k!\,(a-1)!} =$$

$$= \binom{a+k-1}{k} \left(\frac{b}{b+\Lambda(B)}\right)^a \left(\frac{\Lambda(B)}{b+\Lambda(B)}\right)^k$$

... Probabilities of negative binomial distribution with parameters a, $\underbrace{\frac{\Lambda(B)}{b+\Lambda(B)}}_{\text{Prob. of success.}}$

series of Bernoulli trials:

Y = number of successes before ath failure