

10. Determine the Laplace transform of a binomial point process.

$\dots X_1, \dots, X_m$ i.i.d.

$f \geq 0$ measurable function on $\Xi \rightarrow \mathbb{P}(B, \nu, m)$

$$L_{\Phi}(f) = \mathbb{E} \exp \left\{ - \int_{\Xi} f(x) \Phi(dx) \right\} = \mathbb{E} \exp \left\{ - \sum_{i=1}^m f(X_i) \right\} = (*)$$

$\sum_{X \in \text{supp } \Phi} f(X)$... assuming Φ is simple

$$(*) = \mathbb{E} \prod_{i=1}^m \exp \left\{ - f(X_i) \right\} \stackrel{\substack{\text{binomial p.p.} \\ m \text{ sites} \\ \uparrow \text{indep.}}}{=} \prod_{i=1}^m \mathbb{E} \exp \left\{ - f(X_i) \right\} = \left(\mathbb{E} \exp \left\{ - f(X_1) \right\} \right)^m$$

$X_1 \sim \frac{\nu(\cdot)}{\nu(B)}$

(possible because number of points is deterministic)