$ET_n \to 1 \qquad EV_n \to 0$

$$T_n = \prod_{j=1}^{r_n} \left(1 + it Z_{nj}\right)$$

T_n je stejnoměrně integrovatelná posloupnost

$$V_n = \prod_{j=1}^{r_n} \left(1 + it Z_{nj}\right) \left[\exp\left\{ -\frac{t^2}{2} \sum_{j=1}^{r_n} Z_{nj}^2 + \underbrace{\sum_{j=1}^{r_n} r(t Z_{nj})}_{} \right\} - e^{-t^2/2} \right]$$

$r(x^3) \le |x|^3$ pro $|x| < 1$

$$\exp\left(it \sum_{j=1}^{r_n} Z_{nj}\right) = T_n \cdot e^{-t^2/2} + V_n$$

$$V_n = \exp\left(it \sum_{j=1}^{r_n} Z_{nj}\right) - T_n e^{-t^2/2},$$

$$\left|\exp\left(it \sum_{j=1}^{r_n} Z_{nj}\right)\right| \le 1$$

stejnoměrně integrovatelná

$$\lim_{N\to\infty} \sup_{n} E\left|\exp\left(it\sum_{j=1}^{n_n} Z_{nj}\right)\right| \cdot 1[| \quad | > N] = 0$$

$|r(x)| < |x|^3$ pro $|x| < 1$

V_n je stejnoměrně integrovatelná

$E V_n \xrightarrow[n\to\infty]{} 0$ a tomu stačí $V_n \xrightarrow{P} 0$

$$\sum_{j=1}^{n_n} r(tZ_{n,j}) \qquad P\left[\left|\sum r(tZ_{n,j})\right| \geq |t|^3 \sum |Z_{n,j}|^3\right] \to 0$$

$$\max_{j\le n_n}|Z_{n,j}| \leq \max_{j\le n_n}|X_{n,j}| \xrightarrow{P} 0$$

pro n velkém $|tZ_{nj}| < 1$ s velkou pravděpodobností

$$\forall t \in \mathbb{R} \quad P\Big[\max_{j \le n_n} |t Z_{n,j}| > 1\Big] \xrightarrow[n\to\infty]{} 0$$

$$|t|^3 \sum |Z_{n,j}|^3 \le |t|^3 \max |Z_{n,j}| \cdot \underbrace{\sum_{j=1}^{n_n} |Z_{n,j}|^2}_{\downarrow P \; ?} \xrightarrow{P} 0$$

$$\max |Z_{n,j}| \xrightarrow{P} 0$$

$$\le 2 + X_{n,j}^2$$

$$P[X_{n,j}^2 > \varepsilon] \to 0$$

s pravděpodobností jdoucí k 1

$$\Big| \sum r(t Z_{n,j}) \Big| \le |t|^3 \sum_{j=1}^{n_n} |Z_{n,j}|^3 \xrightarrow{P} 0$$

$$\Big| \sum r(t Z_{n,j}) \Big| \xrightarrow{P} 0$$

$$|V_n| = |T_n| \left[\exp\Big\{ -\frac{t^2}{2} \underbrace{\sum Z_{nj}^2}_{\xrightarrow{P} 1} + \underbrace{\sum r(t Z_{nj})}_{\xrightarrow{P} 0} \Big\} - e^{-t^2/2} \right]$$

$$\underbrace{1 \cdot \exp\left(-\frac{t^2}{2}\right) - e^{-t^2/2} \cdot 1}_{} \xrightarrow{P} 0$$

$$|T_n| = \left(\prod_{j=1}^{r_n} (1 + t^2 Z_{nj}^2) \right)^{1/2} \le \left(\prod_{j=1}^{r_n} e^{t^2 Z_{nj}^2} \right)^{1/2} = \exp\left(\frac{t^2}{2} \overbrace{\sum_{j=1}^{r_n} Z_{nj}^2}^{\xrightarrow{P} 1} \right) \xrightarrow{P} e^{t^2/2}$$

$1 + x \le e^x$

$$\underline{|V_n| \xrightarrow{P} 0} \implies$$

díky stejnoměrné integrovatelnosti $E|V_n| \xrightarrow[n\to\infty]{} 0$

$$ET_n \longrightarrow 1 \ ?$$

$$ET_n = E\prod_{j=1}^{r_n}(1+i\vartheta Z_{nj}) = E\left[E\left(\prod_{j=1}^{r_n}(1+i\vartheta Z_{nj}) \,\Big|\, \mathcal{F}_{n,r_n-1}\right)\right]$$

$$= E\left[\prod_{j=1}^{r_n-1}(1+i\vartheta Z_{nj})\, \underbrace{E\left(1+i\vartheta Z_{n,r_n} \,\big|\, \mathcal{F}_{n,r_n-1}\right)}_{1+0=1}\right] = E\left[\prod_{j=1}^{r_n-1}(1+i\vartheta Z_{nj})\right]$$

$$\Big\downarrow \,\big|\, \mathcal{F}_{n,r_n-2}$$

$$= \ \cdots\cdots \ = E\left[\prod_{j=1}^{r_n-2}(1+i\vartheta Z_{nj})\right] = \cdots \ = E(1+i\vartheta Z_{n1}) = 1$$

$$S_n = S_{n,r_n} = \sum_{j=1}^{r_n} X_{n,j} \xrightarrow{d} N(0,1)$$

Zapojme do našich úvah čas

$t \in [0,1]$

$$S^*_{n,t} = \sum_{j=1}^{r_n(t)} X_{n,j}$$

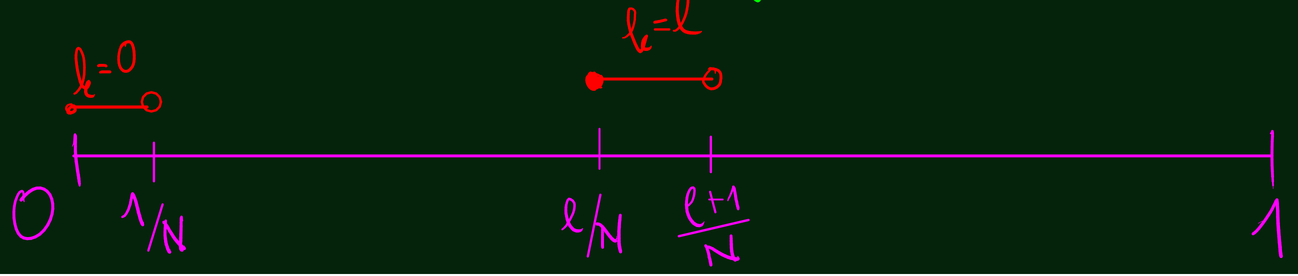
$$r_n(t) = \max\{k;\ k \le t\cdot r_n\}$$

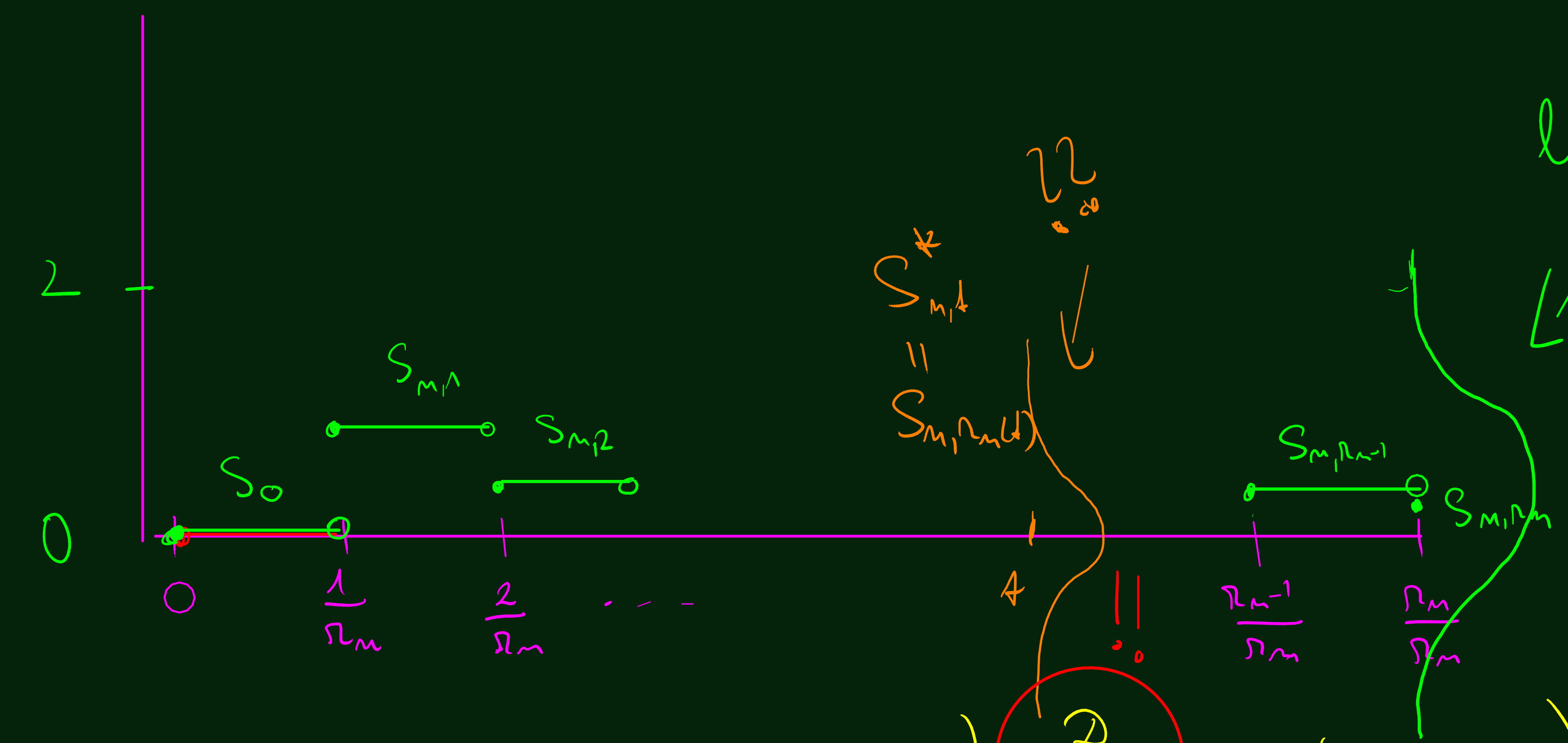
$$\max\left\{k;\ \frac{k}{N} \le t\right\}$$

$r_n = N$

$S_{n,0} = 0$

$$S_{n,l} = \sum_{j=1}^{l} X_{n,j}$$





limitní Gaussovo $N(0,1)$ rozdělení S_{n,r_n}

$$\left(S^*_{n,t},\ t\in(0,1]\right) \xrightarrow{\mathcal{D}} G = \left(G_t,\ t\geq 0\right)$$

zprava spojité po částech konst. procesy