

$\rightarrow X_1, \dots, X_n$ indep. in $\mathcal{B} \sim \nu(\cdot)$

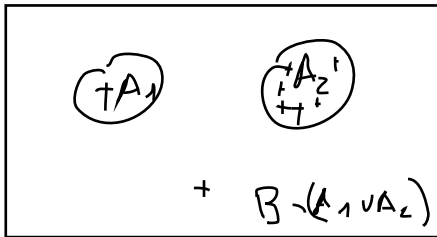
6. Let Φ be a binomial point process with n points in B and the measure ν . Determine the covariance $\text{cov}(\Phi(B_1), \Phi(B_2))$ for $B_1, B_2 \in \mathcal{B}$.

as in previous exercise:

$$\text{cov}(\Phi(B_1), \Phi(B_2)) = \text{cov}(\Phi(B_1 \setminus B_2), \Phi(B_1 \cap B_2)) + \text{cov}(\Phi(B_1 \setminus B_2), \Phi(B_2 \setminus B_1)) + \text{cov}(\Phi(B_1 \cap B_2), \Phi(B_2 \setminus B_1)) + \text{var} \Phi(B_1 \cap B_2)$$

A_1, A_2 disjoint:

$$\text{cov}(\Phi(A_1), \Phi(A_2)) = -m \cdot \frac{\nu(A_1)}{\nu(B)} \cdot \frac{\nu(A_2)}{\nu(B)} \quad \left(\text{var} \Phi(A) = m \frac{\nu(A)}{\nu(B)} \left(1 - \frac{\nu(A)}{\nu(B)}\right) \right)$$



$$(\Phi(A_1), \Phi(A_2), \Phi(B \setminus (A_1 \cup A_2)))$$

$$\sim M\left(m; \left(\frac{\nu(A_1)}{\nu(B)}, \frac{\nu(A_2)}{\nu(B)}, 1 - \frac{\nu(A_1)}{\nu(B)} - \frac{\nu(A_2)}{\nu(B)}\right)\right)$$

$$\dots = -\frac{m}{\nu(B)^2} \left[\underbrace{\nu(B_1 \setminus B_2)} \cdot \underbrace{(\nu(B_1 \cap B_2) + \nu(B_1 \setminus B_2) \nu(B_2 \setminus B_1))}_{\nu(B_2)} + \underbrace{\nu(B_1 \cap B_2)} \cdot \underbrace{\nu(B_2 \setminus B_1)} - \underbrace{\nu(B_1 \cap B_2)} (\nu(B) - \nu(B_1 \cap B_2)) \right] =$$

$$= -\frac{m}{\nu(B)^2} \left[\underbrace{\nu(B_1 \setminus B_2)} \cdot \underbrace{\nu(B_2)} + \underbrace{\nu(B_1 \cap B_2)} \cdot \underbrace{\nu(B_2)} - \nu(B) \nu(B_1 \cap B_2) \right]$$

$$= -\frac{m}{\nu(B)^2} \left[\nu(B_1) \nu(B_2) - \nu(B) \nu(B_1 \cap B_2) \right].$$

