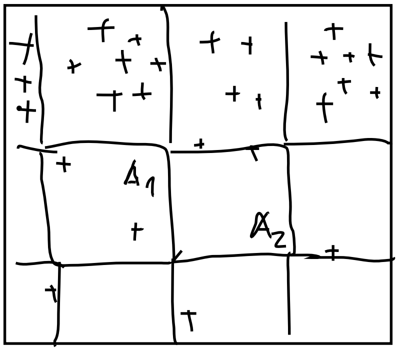
4. Consider the point pattern $\{x_1, \dots, x_n\}$ observed in a compact observation window $W \subset \mathbb{R}^2$. Suggest a test of the null hypothesis that the point pattern is a realization of a Poisson process.



W

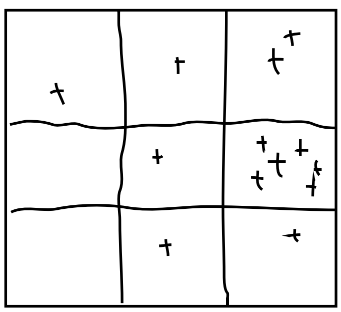
$\left[\ \Phi(A_1) = 2 \sim Po(\Lambda(A_1))\ \right]$

$A_1, \dots, A_\ell$ ... disjoint

$\Downarrow$

$\Phi(A_1), \dots, \Phi(A_\ell)$ indep.

$\sim Po(\Lambda(A_1))$ $\qquad \sim Po(\Lambda(A_\ell))$



regular grid $\Rightarrow$ $|A_1| = \dots = |A_\ell|$

$\not\Rightarrow$ $\Lambda(A_1) = \dots = \Lambda(A_\ell)$

specify H_0: $\{x_1, \dots, x_n\}$ is a realization of a station

Poisson process ... $\Lambda(B) = \lambda \cdot |B|$, λ

(T)

$\Downarrow$ $|A_1| = \dots = |A_\ell| \Rightarrow \Lambda(A_1) = \dots = \Lambda(A_\ell)$ ... ev

λ

$\Rightarrow$ $\Phi(A_1), \dots, \Phi(A_\ell)$ is a random sample

$\Rightarrow$ apply test of Poisson dist. from iid se

if $\lambda(u)$ known: $A_1, \dots, A_\ell$ such that $\Lambda(A_1) = \int_{A_1} \lambda(u$

$\lambda(u; \theta)$ $\qquad = \dots = \Lambda(A_\ell)$ ✓

$|A_1| \neq \dots \neq |A_\ell|$

(T) can test:

I) assume Poisson process, H_0: stationarity

II) assume stationarity, H_0: Poisson process