2. Let Φ be a Poisson point process with the intensity measure Λ and $B \in \mathcal{B}$ be a given Borel set. Show that $\Phi|_B$ is a Poisson point process and determine its intensity measure.

$\hookrightarrow \quad \Phi|_B(A) = \Phi(A \cap B) \qquad \forall A \in \mathcal{B}$

1) Poisson distribution of point counts:

$\Phi(A \cap B) \sim Po(\Lambda(A \cap B$

$A \in \mathcal{B}$: $\quad \mathbb{P}(\Phi|_B(A) = k) = \mathbb{P}(\Phi(A \cap B) = k) =$

$k \in \mathbb{N}_0$

$= e^{-\Lambda(A \cap B)} \cdot \frac{1}{k!} \cdot (\Lambda(A \cap B))^k$

$\hookrightarrow$ intensity measure of $\Phi|_B$ is $\Lambda|_B = \Lambda(\cdot \cap$

2) independence ?

$A_1, \dots, A_k \in \mathcal{B}$ disjoint $\overset{?}{\Rightarrow}$ $\Phi|_B(A_1), \dots, \Phi|_B(A_k)$ inde

$\Downarrow \qquad\qquad \parallel \qquad \parallel$

$A_1 \cap B, \dots, A_k \cap B$ disjoint $\Rightarrow \Phi(A_1 \cap B), \dots, \Phi(A_k \cap B)$ inde

... Φ is P

$\Rightarrow$ YES

$\Rightarrow \Phi|_B$ is Poisson process with int. measure $\Lambda|_B$

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checking void probabilities? only for simple p.p.s

$\Rightarrow$ only if Λ is diffuse