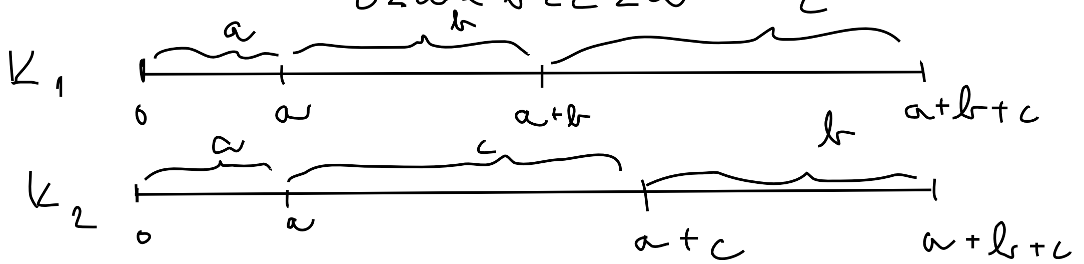
7. For $0 < a < b < c$ let us consider the sets $K_1 = \{0, a, a+b, a+b+c\}$ and $K_2 = \{0, a, a+c, a+b+c\}$. Let X_0 be a random variable with the uniform distribution on the interval $[0, a+b+c]$. We define simple point processes Φ_1 and Φ_2 on $\mathbb{R}$ such that $\operatorname{supp} \Phi_i = \{x \in \mathbb{R} : x = X_0 + y + z(a+b+c), y \in K_i, z \in \mathbb{Z}\}$, $i = 1, 2$. Show that $\mathbb{P}(\Phi_1(I) = 0) = \mathbb{P}(\Phi_2(I) = 0)$ for every interval $I \subseteq \mathbb{R}$ but the distributions of Φ_1 and Φ_2 are different.

$0 < a < b < c < \infty$



$X_0 \sim U[0, a+b$

I) $\mathbb{P}(\Phi_1(I) = 0) = ?$ $\quad$ I interval with length $|I|$

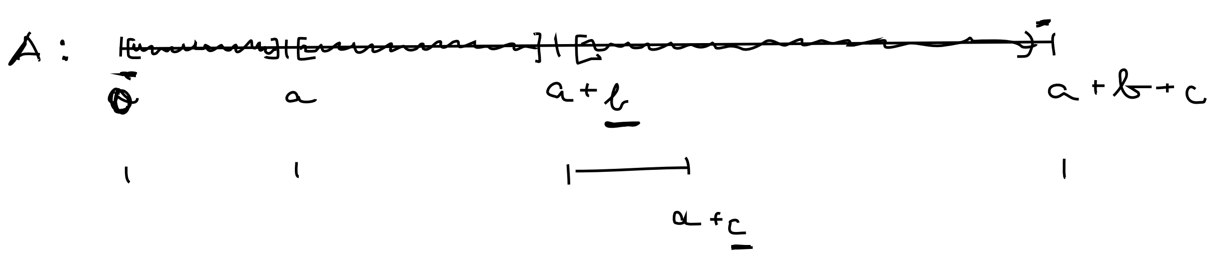
$$= \mathbb{1}(|I| < c) \cdot \frac{1}{a+b+c} \cdot \left((a-|I|)^+ + (b-|I|)^+ + (c-|I|)^+\right)$$

$$\parallel$$

$$\mathbb{P}(\Phi_2(I) = 0) =$$

II) $\mathbb{P}(\Phi_1(A) = 0) \neq \mathbb{P}(\Phi_2(A) = 0)$ for some $A \in \mathcal{B}(\mathbb{R})$

$A = [\varepsilon, a-\varepsilon] \cup [a+\varepsilon, a+b-\varepsilon] \cup [a+b+\varepsilon, a+b+c-\varepsilon]$, $0 < \varepsilon < \frac{1}{2}$



$\mathbb{P}(\Phi_2(A) = 0) = 0$

$\mathbb{P}(\Phi_1(A) = 0) = \dfrac{1}{a+b+c} \cdot (\varepsilon + \varepsilon) > 0$

starting points of A in $(0, \varepsilon)$

(or $X_0 \in (0, \varepsilon)$

starting points of A in $(a+b$

a

(or X_0 in

simple point processes ... all atoms have weight 1