$\underline{7.1}$: $\mathbb{P}(\text{shoda}) = 0{,}42^2 + 0{,}39^2 + 0{,}15^2 + 0{,}04^2 = 0{,}3526$

$\{\text{shoda}\} = \{ (A, A), (0,0), (B,B), (AB, AB)\}$ — p_1, p_2

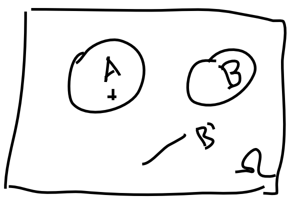
$\Omega = \{ (A,A), (A,B), (A,0), \dots, (AB, AB)\}$ --- 16 jevů

$\mathbb{P}(A_1 \cap A_2) = \mathbb{P}((A,A)) = \mathbb{P}(A_1)\cdot\mathbb{P}(A_2) = 0{,}42^2$

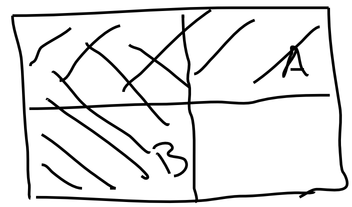
$\mathbb{P}((A,B)) = \mathbb{P}(A)\,\mathbb{P}(B) = 0{,}42 \cdot 0{,}15$ } nezávislost kr… skupin obou partnerů

Hlasovací otázka 6: A, B disj. $\Rightarrow A \cap B = \emptyset \Rightarrow \mathbb{P}(A\cap B) = 0$

NZ? $0 = \mathbb{P}(A\cap B) = \mathbb{P}(A)\cdot\mathbb{P}(B) \iff \mathbb{P}(A) = 0$ nebo $\mathbb{P}(B) = 0$



závislé jevy: „A nastane ⇒ ⇒ B nenastane"



nezávislé jevy

$\mathbb{P}(A) > 0$, $\mathbb{P}(B) > 0$, $A \cap B = \emptyset \Rightarrow A, B$ závislé

$(\mathbb{P}(A) = 0$ nebo $\mathbb{P}(B) = 0)$, $A\cap B = \emptyset \Rightarrow A, B$ nezávislé } $\boxed{C}$

$\underline{\text{Pozn.}}$: $\mathbb{P}(A) = 0$, B libovolný jev $\Rightarrow A, B$ nezávislé

$0 = \mathbb{P}(A\cap B) = \underbrace{\mathbb{P}(A)}_{0} \cdot \underbrace{\mathbb{P}(B)}_{?} = 0$

$A \cap B \subseteq A$

$0 = \mathbb{P}(A\cap B) \leq \mathbb{P}(A) = \dots$

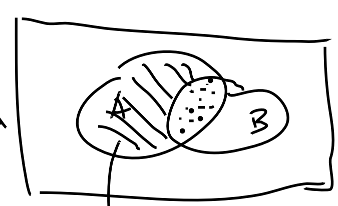
$\underline{\text{Důkaz}}$, že A, B NZ $\Rightarrow A, B^c$ NZ

$\mathbb{P}(A \cap B^c) = \mathbb{P}(A \setminus (A\cap B)) \underset{(A\cap B)\subseteq A}{=}$

$= \mathbb{P}(A) - \mathbb{P}(A\cap B) \underset{\text{NZ } A,B}{=}$

$= \mathbb{P}(A) - \mathbb{P}(A)\mathbb{P}(B) =$

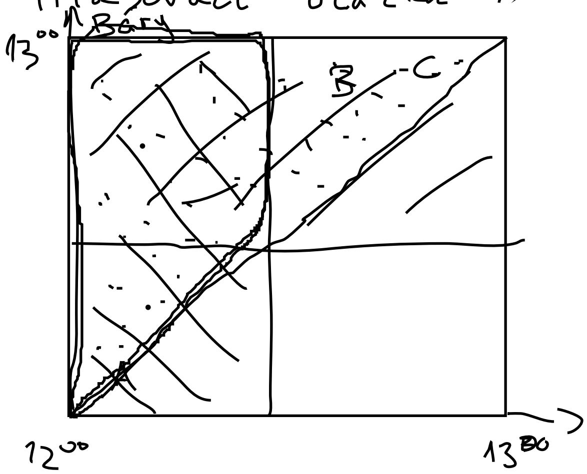
$= \mathbb{P}(A)(1 - \mathbb{P}(B)) = \mathbb{P}(A)\cdot\mathbb{P}(B^c)$ ✓



$\to A \cap B^c = A \setminus (A\cap B)$

□.

Hlasovací otázka 7:



$A = \{A < 12^{30}\}$, $\mathbb{P}(A) = 1/2$

$B = \{B > 12^{30}\}$, $\mathbb{P}(B) = 1/2$

$C = \{A < B\}$, $\mathbb{P}(C) = 1/2$

$\frac{1}{4} = \mathbb{P}(A \cap B) \overset{?}{=} \mathbb{P}(A) \cdot \mathbb{P}(B) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

A, B jsou NZ

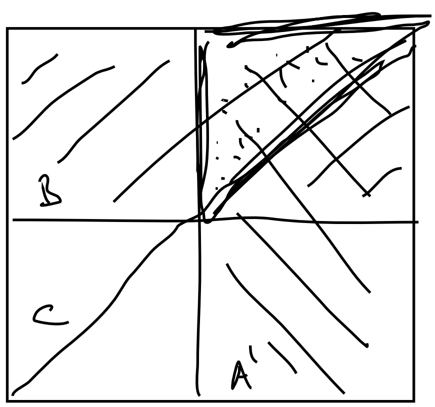
$\frac{3}{8} = \mathbb{P}(A \cap C) \overset{?}{\neq} \mathbb{P}(A) \cdot \mathbb{P}(C) = \frac{1}{4}$, A, C

$\frac{3}{8} = \mathbb{P}(B \cap C) \overset{?}{\neq} \mathbb{P}(B) \cdot \mathbb{P}(C) = \frac{1}{4}$, B, C nejsou NZ

A, B, C nezávislé? NE, protože nejsou všechny dvojice NZ

(resp. $\frac{1}{4} = \mathbb{P}(A \cap B \cap C) \overset{?}{\neq} \mathbb{P}(A) \cdot \mathbb{P}(B) \cdot \mathbb{P}(C) = \frac{1}{8}$

Doplňující otázka: $A' = \{A > 12^{20}\}$



A', B jsou NZ (z obrázku), B, C nejsou

(viz

$\frac{1}{8} = \mathbb{P}(A' \cap B \cap C) = \mathbb{P}(A') \cdot \mathbb{P}(B) \cdot \mathbb{P}(C) = \frac{1}{8}$ ✓

$\mathbb{P}(A' \cap C) = \frac{1}{8} \neq \mathbb{P}(A') \cdot \mathbb{P}(C) = \frac{1}{4}$ ... A', C nejso

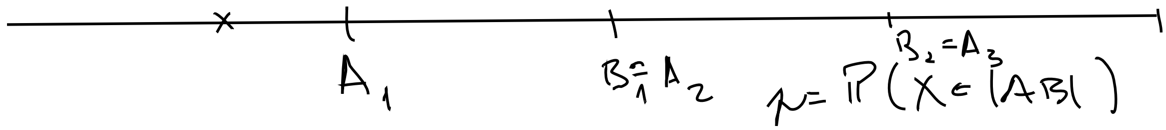
$\Rightarrow$ A', B, C NEJSOU NZ

7.2: N ... celkový počet chyb

odhalil první ... $\frac{a}{N}$ ... odhad psti, že danou chybu

druhý ... $\frac{b}{N}$ ... odhad psti, že danou chyb

odhalili oba ... $\frac{c}{N}$ ... odhad psti, že danou chybu odhalí oba



B_3

$p = \mathbb{P}(X \in [A, B)) \in [0,1]$

předp.: $p > 0$

$\Rightarrow \quad p = \mathbb{P}(X \in [A_i, B_i)) \quad \forall i$

$\exists N \in \mathbb{N}: \quad \mathbb{P}(X \in [A_1, B_N)) = N \cdot p > 1 \qquad \lightning$

dokázali jsme, že na $\mathbb{R}$ neexistuje nenulová translační invariantní míra, která má $\mu(\mathbb{R}) = 1$.