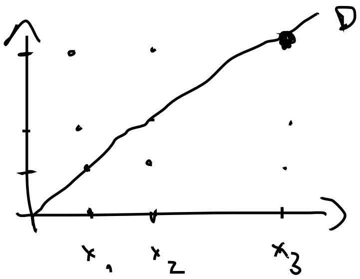
**5.** Why the measure $M^{(n)}$ cannot have a density w.r.t. the Lebesgue measure on $\mathbb{R}^{n\cdot d}$ but the measure $\alpha^{(n)}$ can? Consider $n = 2, d = 1$.

Assume: $\Psi$ atomic (e.g. point process)




non-random $\Psi$: $\Psi \overset{?}{\to} \Psi^{(2)}$

$\hookrightarrow \Psi = \sum_i \delta_{x_i}$

$\Psi^{(2)}(A)$

$A \in \mathcal{B}(\mathbb{R}^2)$

$\Psi^{(2)}(B_1 \times B_2) = \Psi(B$

$B_1, B_2 \in \mathcal{B}(\mathbb{R})$

$= \left(\sum_i \delta_{x_i}(B_1)\right)\Big($

$= \sum_i \sum_j \delta_{x_i}(B_1)\,\delta_{x_j}(B_2) = \sum_i \sum_j \delta_{(x_i, x_j)}(B_1 \times B_2)$

$\delta_{x_i}(B_1)\,\delta_{x_j}(B_2) = \mathbb{1}(x_i \in B_1)\,\mathbb{1}(x_j \in B_2$

$= \mathbb{1}(x_i \in B_1, x_j \in B_2) = \mathbb{1}((x_i, x_j) \in B_1 \times B$

$= \delta_{(x_i, x_j)}(B_1 \times B_2)$

now think about random $\Psi$ and $\mathbb{E}\Psi^{(2)}$ ... can have dens

$D = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 = x_2\}$

$\Psi^{(2)}(D) \geq 0$ , $\mathbb{E}\Psi^{(2)}(D) > 0$ unless $P(\Psi(\mathbb{R}) = 0) = 1$

$0 < M^{(2)}(D) \overset{?}{=} \int_D m(u_1, u_2)\, d(u_1, u_2) = 0$

$\mathrm{Leb}_2(D) = 0$

$\Rightarrow \mathbb{E}^{[2]}$ ... diagonal removed $\Rightarrow \alpha^{(2)}$ may have density w.r.t. 

$\hookrightarrow$ for some processes it do

~~for~~ (some) does not others