4. Let Ψ be a random measure. Check that the following formulas hold for $B, B_1, B_2 \in \mathcal{B}$:

a) $\operatorname{var}\Psi(B) = M^{(2)}(B \times B) - \Lambda(B)^2$,

b) $\operatorname{cov}(\Psi(B_1), \Psi(B_2)) = M^{(2)}(B_1 \times B_2) - \Lambda(B_1)\Lambda(B_2)$.

Def: $M^{(m)}(A) = \mathbb{E}\,\Psi^{(m)}(A)$, $A \in \mathcal{B}(E^m)$ $\quad (E = \mathbb{R}^d)$ $\quad E^m = \mathbb{R}^{d \cdot m}$

Ψ ... random measure

$\hookrightarrow M^{(m)}(A_1 \times \ldots \times A_m) = \mathbb{E}\,\Psi(A_1) \cdot \ldots \cdot \Psi(A_m)$, $A_i \in \mathcal{B}(E)$

Def.: $\alpha^{(m)}(A) = \mathbb{E}\,\Psi^{[m]}(A)$, $A \in \mathcal{B}(E^m)$

$m^{[m]} = \mu^m\big|_{E^{[m]}}$, $E^{[m]} = \{(x_1, \ldots, x_m) \in E^m : x_i \neq$

$\Lambda(A) = \mathbb{E}\,\Psi(A) = M^{(1)}(A) = \alpha^{(1)}(A)$

a) $\operatorname{var}\Psi(B) = \underbrace{\mathbb{E}\,\Psi(B)^2}_{\substack{A_1 = A_2 = B \\ \rightsquigarrow M^{(2)}(A_1 \times A_2) = M^{(2)}(B \times B)}} - (\underbrace{\mathbb{E}\,\Psi(B)}_{\Lambda(B)})^2 = M^{(2)}(B \times B) - \Lambda(B)^2$ ✓

b) $\operatorname{cov}(\Psi(B_1), \Psi(B_2)) = \underbrace{\mathbb{E}\,\Psi(B_1)\Psi(B_2)}_{M^{(2)}(B_1 \times B_2)} - (\underbrace{\mathbb{E}\,\Psi(B_1)}_{\Lambda(B_1)})(\underbrace{\mathbb{E}\,\Psi(B_2)}_{\Lambda(B_2)})$ ✓