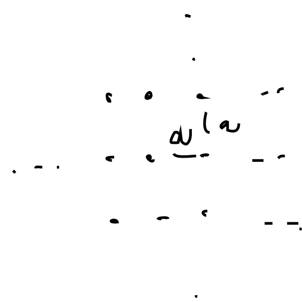
3. Consider intependent random variables U_1 a U_2 with uniform distribution on the interval $[0, a]$, $a > 0$, and the point process Φ on $\mathbb{R}^2$ defined as

$$\Phi = \sum_{m,n \in \mathbb{Z}} \delta_{(U_1+ma, U_2+na)}.$$

Determine the intensity measure of this process.



$$\hookrightarrow \Lambda(B) = \mathbb{E}\,\Phi(B) =$$

$$B \in \mathcal{B}(E) = \mathcal{B}^2$$

$$= \mathbb{E} \sum_{m,n \in \mathbb{Z}} \left(\delta_{(U_1+ma,\, U_2+na)}(B) \right) =$$

$$= \mathbb{E} \sum_{m,n \in \mathbb{Z}} \mathbb{1}_B\big((U_1+ma,\, U_2+na)\big) =$$

$$\hookrightarrow \Sigma$$

$$|A_{m,n}| = a^2$$

$$= \sum_{m,n \in \mathbb{Z}} \mathbb{P}\big(\underbrace{(U_1+ma,\, U_2+na)}_{(X,Y) \sim U[[ma,(m+1)a] \times [na,(n+1)a]]} \in B\big) =$$

$$= A_{m,n}$$

$$= \sum_{m,n \in \mathbb{Z}} \int_{B \cap A_{m,n}} \frac{1}{a^2}\, dx =$$

$B \cap A_{m,n}$ ... subset of $\mathbb{R}^2$

$$= \frac{1}{a^2} \sum_{m,n \in \mathbb{Z}} |B \cap A_{m,n}| = \frac{1}{a^2} \cdot |B| \quad , B \in \mathcal{B}^2$$

$\Lambda(.)$ -- has density w.r.t. 2-dim. Leb measure

$$\Lambda(B) = \frac{1}{a^2}|B| = \int_B \frac{1}{a^2}\, dx$$

$\lambda(w) = \frac{1}{a^2}$, $w \in \mathbb{R}^2$ ... intensity function

$$\hookrightarrow \lambda(w)|dw| \approx \mathbb{P}(\Phi(dw) \geq 1)$$

$$\hookrightarrow \lim_{|dw| \to 0} \frac{\mathbb{P}(\Phi(dw) \geq 1)}{|dw|}$$