Ψ_B

2. Prove that Ψ is a random measure if and only if $\Psi(B)$ is a random variable for every $B \in \mathcal{B}$.

$\Psi_B(\omega)$ $(\Omega, \mathcal{A}, \mathbb{P})$

"$\Rightarrow$" $\Psi(B) = \pi_B(\Psi)$ .... $[\Psi(B)](\omega) = \pi_B(\Psi(\omega))$

B fixed $\Psi(B) = \pi_B \circ \Psi$ ... composition of measurable mapp

↳ see 3.1 a) ↳ assumed

"$\Leftarrow$" $\Psi: (\Omega, \mathcal{A}, \mathbb{P}) \to (\mathcal{U}, \mathcal{H})$ ... measurable ?

$\mathcal{U}_{B,r} \in \mathcal{H}$, $\Psi^{-1}(\mathcal{U}_{B,r}) = \{\omega \in \Omega : \Psi(\omega) \in \mathcal{U}_{B,r}\} =$

$= \{\omega \in \Omega : [\Psi(\omega)](B) < r\} =$

$= \{\omega \in \Omega : [\Psi(B)](\omega) < r\} \in \mathcal{A}$ ↔ $[\Psi(B)](\omega)$

$= [\Psi(B)]^{-1}(\langle 0, r \rangle)$ ... we assume $\Psi(B)$ measurable