6. Discuss how to estimate the semivariogram of an isotropic intrinsic stationary random field, based on the observations $Z(x_1), \dots, Z(x_n)$.

7. Discuss how to test the independence of two stationary random fields defined on the same domain, based on the observations $(\underline{Z_1(x_1)}, Z_2(x_1)), \dots, (\underline{Z_1(x_n)}, Z_2(x_n))$.

↳ hint: assume that values $Z_2(x)$ a
observed for $\forall$

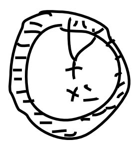
ad 6)

for intrinsic stationary RF:

$$\underline{\gamma(h)} = \frac{1}{2} \operatorname{var}\left(Z(x) - Z(y)\right), \quad h = \|x - y\|$$

$x, y \in \mathbb{R}^d$

observed: $Z(x_1), \dots, Z(x_n)$, $x_1, \dots, x_n \in \mathbb{R}^d$



$\|x_1 - x_2\|$

$\|x_1 - x_3\|$

...

$\hat{\gamma}(h)$ ... use pairs of points with $h - \varepsilon \le \|x_i - x_j\| \le h + \varepsilon$

$h > 0$

$$\hat{\gamma}(h) = \sum_i \sum_{j=1}^{n} \frac{1}{2} \frac{1}{N(h)} \left(Z(x_i) - Z(x_j)\right)^2 \mathbb{1}\left(h - \varepsilon \le \|x_i - x_j\| \le h +\right.$$

$\hat{\gamma}(h_1), \dots, \hat{\gamma}(h_k)$

$N(h)$ ... number of pairs with

basic form of kernel estimation