2. Let $\{W^H(t) : t \in \mathbb{R}^d_+\}$ be a centered Gaussian random field with the covariances

$$\mathrm{cov}(W^H(t), W^H(s))\ \mathsf{E}\, W^H(t)W^H(s) = \frac{1}{2}(\underbrace{\|t\|^{2H} + \|s\|^{2H}} - \|\underbrace{t}_{0} - s\|^{2H}),\ t, s \in \mathbb{R}^d_+, \qquad \mathrm{var}\, W^H(t) = \|t\|^{2H}\ \forall t$$

where $H \in (0, 1)$. Such a random field is called the *Lévy's fractional Brownian random field.* Show that it is an intrinsically stationary random field and determine its variogram.

$H = \frac{1}{2}$ ... Brownian RF

intrinsic stationary: 1) $\mathbb{E}(W^H(s) - W^H(t)) = 0 \quad \forall s, t \in \mathbb{R}$

2) $\mathrm{var}(W^H(s) - W^H(t))$ is a function of $t-s$ only.

ad 1) $W^H(t)$ is centered ✓

ad 2) $\mathrm{var}(W^H(s) - W^H(t)) = \mathrm{var}\, W^H(s) + \mathrm{var}\, W^H(t) -$

$\forall s, t \in \mathbb{R}_0^+$

$- 2\,\mathrm{cov}(W^H(s), W^H(t)) =$

$= \|s\|^{2H} + \|t\|^{2H} - (\|t\|^{2H} + \|s\|^{2H} - \|t-s\|^{2H}) = \|t-s\|^{2H}$

$\Rightarrow \{W^H(t), t \in \mathbb{R}_0^+\}$ is intrinsic stationary
(vnitřně stac.)

variogram: $2\gamma(h) = \mathrm{var}(W^H(t+h) - W^H(t)) = \|h\|^{2H}, \quad h \in \mathbb{R}^d_+$

semivariogram: $\gamma(h) = \frac{1}{2}\|h\|^{2H}, h \in \mathbb{R}^d_+$.

assume: $\{Z(t), t \in \mathbb{R}^d\}$ weakly stationary ... $C(h) = \mathrm{cov}(Z(t+h), Z$

$2\gamma(h) = \mathrm{var}(Z(t+h) - Z(t)) = \mathrm{var}(Z(t+h)) + \mathrm{var}(Z(t)) -$

$- 2\,\mathrm{cov}(Z(t+h), Z(t)) = C(0) + C(0) - 2 \cdot C(h) =$

$= 2C(0) - 2C(h)$

$\Rightarrow \gamma(h) = C(0) - C(h)$