NMFM402 – Mathematics of Non-Life Insurance 2

GLM 1 - exponential dispersion family (EDF), link function, MLE

Practical 2

To review the necessary theory for this practicals, you may check the lecture notes on Moodle, [1], Chapters 2.1. and 2.2 (GLM - introduction and estimation of parameters). For further reading, see [2], Chapters 2 and 3.

Exercise 1:
Consider a random variable Y having Poisson distribution with parameter $\lambda > 0$, i.e. its PMF is

$$f(y|\lambda) = \frac{\lambda^y \mathrm{e}^{-\lambda}}{y!}, \quad y = 0, 1, 2, ...$$

(a) Verify that the Poisson distribution belongs to the exponential dispersion family (EDF). Find cumulant function b and function c. Consider the weight $w = 1$ and determine parameters θ and φ.

(b) Determine $\mathbb{E}Y$ and $\text{var}Y$ using the cumulant function b.

(c) Find the formula for the variance function $v(\mu)$

(d) Find the canonical link function $g(z)$

Exercise 2:
Consider a GLM with independent random variables $Y_1, ..., Y_n$ having Poisson distribution with expectations satisfying $g(\mathbb{E}Y_i) = \sum_{j=0}^k x_{ij}\beta_j$, where the link function is logarithmic $g(z) = \log(z)$.

(a) Write the expression for log-likelihood for parameters θ_i from EDF model.

(b) Write the expression for log-likelihood for structural parameters β_j from GLM.

(c) Write the equations for MLE of parameters β_j. Firstly, derive the equations from (b). Secondly, obtain the equations by direct use of formula (2.14) in [1].

Exercise 3:
Consider a random variable Y having Gamma distribution with parameters $\alpha > 0$ and $\beta > 0$, i.e. its PDF is

$$f(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \mathrm{e}^{-\beta y}, \quad y > 0.$$

Do the tasks (a)-(d) from Exercise 1.

Exercise 4:
Consider a GLM with independent random variables $Y_1, ..., Y_n$ having Gamma distribution with expectations satisfying $g(\mathbb{E}Y_i) = \sum_{j=0}^k x_{ij}\beta_j$, where the link function is logarithmic $g(z) = \log(z)$. (Note that this link is not canonical for Gamma distribution).
Perform the tasks (a)-(c) from Exercise 2.

Exercise 5 (see Chapter 2.1.4 in [2] for details):
A family of distributions is called *scale invariant*, if for a random variable Y having distribution from this family and arbitrary positive constant $c > 0$, the distribution of cY falls into the same family. Scale invariant distributions are particularly useful in tariffication (we want our probabilistic model to be invariant under changes of monetary units, or exchanging per cent for

per mille etc.). It can be shown that the only distributions from EDFs that are scale invariant are the so called **Tweedie models**, which are defined as having variance function

$$v(\mu) = \mu^p,$$

for some p. Decide, whether the following EDF distributions:

(a) normal,

(b) Poisson,

(c) gamma,

fall into the class of Tweedie models.

Reference

[1] L. Mazurová *Mathematics of Non-life Insurance 2 - lecture notes.*Version March 2021. Available online at Moodle: https://dl1.cuni.cz/pluginfile.php/1162656/mod_resource/content/2/MNP2LectureNotes.pdf

[2] E. Ohlsson, B. Johansson: *Non-Life Insurance Pricing with Generalized Linear Models*, 15 EAA Lecture Notes, DOI 10.1007/978-3-642-10791-7_2, Springer-Verlag Berlin Heidelberg, 2010