Now for the magic numbers: 715; 364; and 924. Let x be the selected unknown number, and a, b, c the respective remainders when divided by 7, 11, and 13. Then

$$\begin{aligned} x &\equiv a \mod 7 \quad \text{and} \quad 715x \equiv 715a \mod 7 \\ x &\equiv b \mod 11 \quad \text{and} \quad 364x \equiv 364b \mod 11 \\ x &\equiv c \mod 13 \quad \text{and} \quad 924x \equiv 924c \mod 13. \end{aligned}$$

Transposing:

$$715(x-a) \equiv 0 \mod 7 \tag{1}$$
$$364(x-b) \equiv 0 \mod 11 \tag{2}$$
$$924(x-c) \equiv 0 \mod 13 \tag{3}$$

But congruence (1) is divisible by 11 and by 13 and therefore by 11, 13, and 7, or 1001; congruence (2) is divisible by 7 and by 13 and therefore by 7, 13, and 11, or 1001; and finally, congruence (3) is divisible by 7 and by 11 and therefore by 7, 11, and 13, or 1001. Adding, we have to the new modulus 1001:

$$2003x-(715a+364b+924c) \equiv 0 \mod 1001$$

or

$$x \equiv 715a+364b+924c \mod 1001.$$

We have thus found the value of x in terms of the three magic numbers and the three remainders a, b, and c, and since it must not exceed 1000 we drop multiples of 1001 from the right-hand side of the congruence so that a solution below this limit is obtained.

A slightly different method of performing this trick is to remember the numbers 5, 4, -1 and the numbers $143 = 11\cdot 13$; $91 = 7\cdot 13$; and $77 = 7\cdot 11$. For the given remainders a, b, c, find the least residues (remainders) of $5a$ mod 7; $4b$ mod 11; and $-c$ mod 13. Multiply 143, 91, and 77 respectively by these residues, add the products, and subtract multiples of 1001. Thus for the remainders 5, 6, 3, we have

$$\begin{aligned} 5\cdot 5 &\equiv 4 \mod 7 \\ 4\cdot 6 &\equiv 2 \mod 11 \\ (-1)\cdot 3 &\equiv -3 \mod 13. \end{aligned}$$

Multiplying 143, 91, and 77 by the respective residues 4, 2, -3, and adding, we have $572+182-231 = 523$. Some may prefer this method because of the smaller numbers involved; it may be easier to keep in mind the two sets 5, 4, -1 and 143, 91, and 77 than the larger numbers, 715, 364, and 924. The reader may now wish to test his grasp of the congruence method by actually deriving the magic numbers, instead of assuming them as above.†