VEKTOROVÁ ALGEBRA

A, B, C – VEKTORY $\qquad A = (a_1, a_2, a_3)$ atd.

f, g – skaláry

$$A.B = a_1 b_1 + a_2 b_2 + a_3 b_3 = a_i b_i$$

$$A \times B = (a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1)$$

$$A.B = B.A \qquad A \times B = -B \times A$$

$$f(A.B) = (fA).B = A.(fB) \qquad f(A \times B) = (fA) \times B = A \times (fB)$$

$$A(B.C) \neq (A.B)C \qquad A \times (B \times C) \neq (A \times B) \times C$$

$$A.(B+C) = A.B + A.C \qquad A \times (B+C) = A \times B + A \times C$$

$$A.B = 0 \qquad A \perp B$$

$$A \times B = 0 \qquad A \parallel B$$

$$A.A = |A|^2 \qquad A \times A = 0$$

$$A \times (B \times C) = B(A.C) - C(A.B)$$

$$A.(B \times C) = C.(A \times B) = B.(C \times A)$$

$$(A \times B).(C \times D) = (A.C)(B.D) - (B.C)(A.D)$$

$$(A \times B)^2 = |A|^2 |B|^2 - (A.B)^2$$

VEKTOROVÝ SOUČIN POMOCÍ LEVI-CIVITOVA TENZORU

$$\varepsilon_{ijk} = \begin{cases} 1 & i,j,k \text{ – sudá permutace čísel } 1,2,3 \ (123 - 312 - 231) \\ 0 & \text{alespoň dva indexy stejné} \\ -1 & ijk \text{ – liché permutace } 321 - 132 - 213 \end{cases}$$

$$\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij} = -\varepsilon_{jik} = -\varepsilon_{ikj} = -\varepsilon_{kji}$$

$$(A \times B)_i = \varepsilon_{ijk} A_j B_k \quad \text{– } i\text{-tá složka součinu}$$

OPERÁTORY – 1

Hamiltonův operátor $\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$

A, B, C – VEKTOROVÁ POLE

f, g – skalární funkce

$\nabla \cdot A = \operatorname{div} A$

$\nabla \times A = \operatorname{rot} A$

$\nabla f = \operatorname{grad} f$

Pozor ! $\nabla f \neq f \nabla \Rightarrow$ OPERÁTOR $\rightarrow \left(f\frac{\partial}{\partial x_1}, f\frac{\partial}{\partial x_2}, f\frac{\partial}{\partial x_3}\right)$

VEKTOR $\rightarrow \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right)$

$\nabla \cdot A \neq A \cdot \nabla \longrightarrow a_i \frac{\partial}{\partial x_i} = a_1 \frac{\partial}{\partial x_1} + a_2 \frac{\partial}{\partial x_2} + a_3 \frac{\partial}{\partial x_3} =$ operátor

$\frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \frac{\partial a_3}{\partial x_3} = \frac{\partial a_i}{\partial x_i} =$ skalár

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TENZOR IDENTITY $\overleftrightarrow{I}$

$\overleftrightarrow{I} \cdot A = A$

$\underline{\nabla \vec{r} = \overleftrightarrow{I}}$

$A \cdot \operatorname{grad} \vec{r} = A \cdot \nabla \vec{r}$

k-tá složka

$A \cdot \nabla \vec{r} \,|_k = (A \cdot \nabla)\, \vec{r}\,|_k = \left(a_j \frac{\partial}{\partial x_j}\right) x_k =$

$= a_j \frac{\partial x_k}{\partial x_j} = a_j \delta_{kj} = a_k$

$\underline{A \cdot \nabla \vec{r} = A}$

$\operatorname{grad} \vec{r} = \overleftrightarrow{I}$

$\operatorname{div} \vec{r} = n$ – dimenze

$\operatorname{rot} \vec{r} = \emptyset$

OPERÁTORY 2

2. derivace

$\nabla \times \nabla = \emptyset$ $\qquad\qquad$ $\nabla \cdot \nabla = \nabla^2 = \Delta$

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

div grad $f = \nabla \cdot (\nabla f) = (\nabla \cdot \nabla) f = \Delta f$

rot grad $f = \nabla \times (\nabla f) = (\nabla \times \nabla) f = \emptyset$

div rot $A = \nabla \cdot (\nabla \times A) = A \cdot (\nabla \times \nabla) = \emptyset$

div grad $A = (\nabla \cdot \nabla) A = \Delta A$ - vekt. pole

rot rot $A = \nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - (\nabla \cdot \nabla) A = \nabla (\nabla \cdot A) - \Delta A$

$\Rightarrow \Delta A =$ grad div A $-$ rot rot A

IDENTITY

konstanta z hlediska derivací

grad $(fg) = \nabla (fg) = \nabla (f_c g) + \nabla (f g_c) = f \nabla g + g \nabla f$

grad $(f(g(\vec{r}))) = \frac{df}{dg}$ grad g

Pozor! $(\nabla f) \times (\nabla g) \neq \emptyset$ $\qquad$ jen pro $f = g$

grad $(A \cdot B) = \nabla (A \cdot B) = \nabla (A_c \cdot B) + \nabla (A \cdot B_c)$

ale lze ukázat že

$A \times (\nabla \times B) = \nabla (A_c \cdot B) - (A \cdot \nabla) B \Rightarrow \nabla (A_c \cdot B) = A \times (\nabla \times B) + (A \cdot \nabla) B$

a podobně

$\nabla (B_c \cdot A) = B \times (\nabla \times A) + (B \cdot \nabla) A$

Po dosazení pak

grad $(A \cdot B) = (A \cdot \nabla) B + (B \cdot \nabla) A + A \times$ rot $B + B \times$ rot A

$A \times$ rot $A = A \times (\nabla \times A) = \nabla (A_c \cdot A) - (A \cdot \nabla) A$

$\nabla (A \cdot A) = \nabla (A_c \cdot A) + \nabla (A \cdot A_c) \Rightarrow \nabla (A_c \cdot A) = \frac{1}{2} \nabla (A \cdot A)$

Po dosazení

$A \times$ rot $A = \frac{1}{2}$ grad $(A \cdot A) - (A \cdot \nabla) A$

OPERATORI 5

$$\operatorname{div}(fA) = \nabla\cdot(fA) = \nabla\cdot(f_c A) + \nabla\cdot(f A_c) =$$
$$= f\,\nabla\cdot A + \nabla f\cdot A_c = f \operatorname{div} A + A\cdot \operatorname{grad} f$$

$$\operatorname{div}(A\times B) = \nabla\cdot(A\times B) = \nabla\cdot(A_c\times B) + \nabla\cdot(A\times B_c) =$$
$$= -A\cdot(\nabla\times B) + B\cdot(\nabla\times A) = B\cdot\operatorname{rot} A - A\cdot\operatorname{rot} B$$

$$\operatorname{rot}(fA) = \nabla\times(fA) = \nabla\times(f_c A) + \nabla\times(f A_c) =$$
$$= f(\nabla\times A) + \nabla f\times A_c = f(\nabla\times A) - A\times\nabla f =$$
$$= f\operatorname{rot} A - A\times\operatorname{grad} f$$

$$\operatorname{rot}(A\times B) = \nabla\times(A\times B) = \nabla\times(A_c\times B) + \nabla\times(A\times B_c)$$
$$\overset{\text{BAC-CAB}}{=} A(\nabla\cdot B) - (A\cdot\nabla)B + (B\cdot\nabla)A - B(\nabla\cdot A)$$
$$= (B\cdot\nabla)A - (A\cdot\nabla)B + A\operatorname{div} B - B\operatorname{div} A$$