1) 80% → min 11 x

ÚČAST - AKTIVNÍ

2) ZÁPOČTOVÝ TEST

∇ $\quad T = T(x,y,z) \qquad T(B)$

$T(A) \quad \vec{A}' = \vec{A} + \underbrace{(dx, dy, dz)}_{d\vec{l}}$

$$T(A') = T(A) + \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz = T(A) + \nabla T \cdot d\vec{l}$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$T(B) = T(A) + \int_A^B \nabla T \cdot d\vec{l}$$

- $$\nabla r = \nabla\left(\sqrt{x^2+y^2+z^2}\right) = \left(\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2x, \ldots\right) = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right) = \frac{\vec{r}}{r}$$

- $$\nabla \frac{1}{r} = \nabla\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right) \Big/_x = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}} \cdot 2x = \frac{-x}{r^3}$$

$$\boxed{\nabla \frac{1}{r} = -\frac{\vec{r}}{r^3}} \sim \frac{1}{r^2}$$

- $$\nabla \frac{\vec{p} \cdot \vec{r}}{r^3} = \nabla \frac{p_x x + p_y y + p_z z}{(x^2+y^2+z^3)^{3/2}}$$

$$\Big/_x = \frac{P_x}{r^3} - \frac{\vec{P}\cdot\vec{r} \cdot \frac{3}{2} \cdot 2x}{(x^2+y^2+z^2)^{5/2}} =$$

$$= \frac{P_x}{r^3} - \frac{3x}{r^5}(\vec{P}\cdot\vec{r})$$

$$\nabla \ldots = \frac{\vec{P}}{r^3} - \frac{3\vec{r}}{r^5}(\vec{P}\cdot\vec{r})$$

$$\nabla\cdot\vec{A} \equiv \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \text{DIV}$$

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \ldots\right)$$

ROT

- $\nabla\cdot\vec{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$

- $\boxed{\text{DÚ} \quad \nabla\cdot\left(\frac{\vec{r}}{r^3}\right)}$

- $\nabla\times\frac{\vec{r}}{r^3}\Big/_x = \frac{\partial}{\partial y}\frac{z}{(x^2+y^2+z^2)^{3/2}} - \frac{\partial}{\partial z}\frac{y}{(x^2+y^2+z^2)^{3/2}}$

$$= \frac{-\frac{3}{2}\cdot z\cdot 2y}{(x^2+y^2+z^2)^{\frac{5}{2}}} - \frac{-\frac{3}{2}y\cdot 2z}{(x^2+y^2+z^2)^{\frac{5}{2}}} = 0$$

$$= (0,0,0) = \vec{0}$$

$$\nabla\cdot\nabla T = \nabla\cdot\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial T^2}{\partial z^2}$$

$$\equiv \Delta T$$

$$\nabla\cdot\nabla \equiv \Delta \quad (\text{LAPLACE}) \qquad \nabla^2$$

$$\nabla \times \nabla T = \nabla \times \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z} \right) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} T$$

$$\left(\frac{\partial}{\partial y} \frac{\partial T}{\partial z} - \frac{\partial \partial T}{\partial z \partial y}, \frac{\partial^2 T}{\partial z \partial x} - \frac{\partial^2 T}{\partial x \partial z} \right) \dots \Big)$$

$$= (0, 0, 0) = \vec{0} \qquad \nabla \times \nabla \equiv \vec{0}$$

$$\nabla \cdot (\nabla \times \vec{A}) = \nabla \cdot \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} =$$

$$= \cancel{\frac{\partial^2 A_z}{\partial x \partial y}} - \cancel{\frac{\partial^2 A_y}{\partial x \partial z}} + \cancel{\frac{\partial^2 A_x}{\partial z \partial y}} - \cancel{\frac{\partial^2 A_z}{\partial x \partial y}} + \cancel{\frac{\partial^2 A_y}{\partial x \partial z}} - \cancel{\frac{\partial^2 A_x}{\partial y \partial z}}$$

$$= 0$$

	∇	$\nabla \cdot$	$\nabla \times$
∇	tenzor		tenzor
$\nabla \cdot$	Δ	N	0
$\nabla \times$	$\vec{0}$	N	

$$\nabla \nabla T = \nabla \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z} \right).$$

$$\nabla (\nabla \cdot \vec{A}) = \nabla \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) =$$

$$\left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial x \partial y} + \frac{\partial^2 A_z}{\partial x \partial z}, \dots, \dots \right)$$

$$\nabla \times \nabla \times \vec{A}$$

$$\vec{A} \times \vec{B} \times \vec{C} = \vec{B} (\vec{A} \cdot \vec{C}) - (\vec{A} \cdot \vec{B}) \vec{C}$$

$$= \nabla (\nabla \cdot \vec{A}) - \underbrace{(\nabla \cdot \nabla) \vec{A}}_{\Delta \vec{A}}$$

$$\Delta \vec{A} = \Delta (A_x, A_y, A_z) = (\Delta A_x, \Delta A_y, \Delta A_z)$$

$$\Delta A_y = \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2}$$