1. Working with cliques:

a) How many cliques are there on the given graph?

b) Which neighbourhood relation on 13 vertices would result in the least possible number of cliques? What is the number of cliques in that case?

c) Which neighbourhood relation on 13 vertices would result in the maximum possible number of cliques? What is the number of cliques in that case?

d) Draw a neighbourhood relation that results in exactly 20 cliques.

e) Is there any other way of representing the neighbourhood relation, other than an undirected graph?

f) Is there any neighbourhood relation relevant for the regions of Czech Republic other than the one based on the common boundary?

a) $\emptyset$, $13\times\{i\}$, $24\times\{i,j\}$, $12\times\{i,j,k\}$ $\Rightarrow$ 50 cliques

b) $\emptyset$, $13\times\{i\}$ $\Rightarrow$ 14

c) $\frac{13\cdot 12}{2} = \binom{13}{2} \times \{i,j\}$ ... number of edges in complete graph

$2^{13} = 8196$ ... number of cliques ... $2^{|L|}$ ... total number of subsets of L

$\binom{13}{0}+\binom{13}{1}+\binom{13}{2}+\binom{13}{3}+\dots+\binom{13}{13} = (1+1)^{13} = 2^{13}$

$\{i,j\}$, $\{i,j,k\}$ → $\{i,j\}$, → $\{j,k\}$, → $\{i,k\}$, $\{i\}, \{j\}, \{k\}, \emptyset$

C clique, $A \subseteq C$ is also a clique

e) matrix representation $A = (a_{ij})_{i,j=1}^{n}$ $a_{ij} = \begin{cases} 1 & \dots\ i\sim j \\ 0 & \dots\ i \nsim j \end{cases}$

f) Euclidean distance, „temporal distance"