

- ΖΗΤΗΣΕΙΣ 1) ΔΟΥΧΙΑΚΕΝΑ 80% 17 mi
 ΑΓΙΩΝΙΣ ΎΠΑΣΤ
 2) ΖΗΡΟΤΟΝΥ ΤΕΣΤ
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$$\nabla T = T(x, y, z)$$

$$\begin{aligned}
 T(A') &= T(A) + \left\{ \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz \right\} \\
 &= \nabla T \cdot d\vec{r}
 \end{aligned}$$

$d\vec{r} = (dx, dy, dz)$
 $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$

$$T(B) = T(A) + \int_A^B \nabla T \cdot d\vec{r}$$

$$\begin{aligned}
 \bullet \nabla R &= \nabla (\sqrt{x^2 + y^2 + z^2}) = \left(\frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + \dots}} \cdot 2x, \dots \right) = \\
 &= \left(\frac{x}{R}, \frac{y}{R}, \frac{z}{R} \right) = \frac{\vec{r}}{R}
 \end{aligned}$$

$$\begin{aligned}
 \bullet \nabla \frac{1}{R} &= \nabla \frac{1}{\sqrt{x^2 + y^2 + z^2}} = \dots \rightarrow \frac{-1}{R^3} (x, y, z) = \frac{-\vec{r}}{R^3} \\
 \nabla_x &= -\frac{x}{R \cdot R^2} = -\frac{x}{R^3}
 \end{aligned}$$

$$\begin{aligned}
 \bullet \nabla \frac{\vec{p} \cdot \vec{r}}{R^3} &= \nabla \frac{p_x x + p_y y + p_z z}{(x^2 + y^2 + z^2)^{3/2}} \\
 \nabla_x &= \frac{p_x}{R^3} - \frac{\vec{p} \cdot \vec{r} \cdot \frac{3}{2} (x^2 + y^2 + z^2)^{1/2} \cdot 2x}{R^6} \\
 &= \frac{p_x}{R^3} - \frac{3 \cdot R x}{R^6} \vec{p} \cdot \vec{r} = \frac{p_x}{R^3} - \frac{3x}{R^5} \vec{p} \cdot \vec{r}
 \end{aligned}$$

$$\nabla \dots = \frac{1}{r^3} - \frac{3r}{r^5} \vec{r} \cdot \vec{r}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad \text{DIV.}$$

$$\nabla \times \vec{A} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{pmatrix} = \begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ \dots \end{pmatrix} \quad \text{CURL}$$

$$\nabla \cdot \vec{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$$

$$\text{DU: } \nabla \cdot \left(\frac{\vec{r}}{r^3} \right)$$

$$\nabla \times \frac{\vec{r}}{r^3} \Big|_x = \left(\frac{\partial}{\partial y} \frac{z}{r^3} - \frac{\partial}{\partial z} \frac{y}{r^3} \right)$$

$$\begin{aligned} &= \frac{\partial}{\partial y} \frac{z}{(x^2+y^2+z^2)^{3/2}} - \frac{\partial}{\partial z} \frac{y}{(x^2+y^2+z^2)^{3/2}} = \\ &= - \frac{z}{r^6} \cdot \frac{3y}{2} + \frac{3y \cdot z \cdot r}{r^6} = \\ &= \frac{-3yz + 3yz}{r^6} = 0 \\ &\nabla \times \frac{\vec{r}}{r^3} = \vec{0} = (0, 0, 0) \end{aligned}$$

$$\nabla \cdot \nabla T = \nabla \cdot \left(\frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \right) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$\equiv \Delta T \quad \text{LAPLACE}$$

$$\nabla^2 T$$

$$\nabla \times \nabla T = \nabla \times \left(\frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 T}{\partial y \partial z} & -\frac{\partial^2 T}{\partial z \partial y} \\ \frac{\partial^2 T}{\partial z \partial x} & -\frac{\partial^2 T}{\partial x \partial z} \\ \frac{\partial^2 T}{\partial x \partial y} & -\frac{\partial^2 T}{\partial y \partial x} \end{vmatrix} = 0$$

$$\nabla \times \nabla = \vec{0}$$

$$\nabla \cdot (\nabla \times \vec{A}) = \nabla \cdot \left(\frac{\partial A_z}{\partial y} \hat{i} - \frac{\partial A_y}{\partial z} \hat{j} + \dots \right)$$

$$= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_x}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} = 0$$

$$\nabla \cdot (\nabla \times) = 0$$

$$\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \vec{A} (\nabla \cdot \nabla)$$

$$= \nabla (\nabla \cdot \vec{A}) - \Delta \vec{A}$$

$$\vec{A} \times \vec{B} \times \vec{C} = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{B} \cdot \vec{A})$$