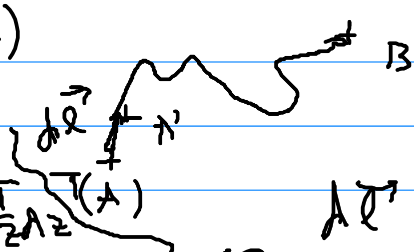
ZÁPOČET 1) DOCHÁZKA 80% 11 mi–

AKTIVNÍ ÚČAST

2) ZÁPOČTOVÝ TEST

∇ $\quad T = T(x, y, z)$



$$T(A') = T(A) + \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz = \nabla T \cdot d\vec{l}$$

$$d\vec{l} = (dx, dy, dz)$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$T(B) = T(A) + \int_A^B \nabla T \cdot d\vec{l}$$

- $$\nabla r = \nabla\left(\sqrt{x^2+y^2+z^2}\right) = \left(\frac{1}{2}\cdot\frac{1}{\sqrt{x^2+\dots}}\cdot 2x, \dots\right) = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right) = \frac{\vec{r}}{r}$$

- $$\nabla\frac{1}{r} = \nabla\frac{1}{\sqrt{x^2+y^2+z^2}} = \dots \quad -\frac{1}{r^3}(x, y, z) = \frac{-\vec{r}}{r^3}$$

$$\nabla_x = -\frac{x}{r\cdot r^2} = -\frac{x}{r^3}$$

- $$\nabla\frac{\vec{p}\cdot\vec{r}}{r^3} = \nabla\frac{p_x x + p_y y + p_z z}{(x^2+y^2+z^2)^{3/2}}$$

$$\nabla_x = \frac{p_x}{r^3} - \frac{\vec{p}\cdot\vec{r}\,\frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\cdot 2x}{r^6}$$

$$= \frac{p_x}{r^3} - \frac{3\cdot r x}{r^6}\,\vec{p}\cdot\vec{r} = \frac{p_x}{r^3} - \frac{3x}{r^5}\,\vec{p}\cdot\vec{r}$$

$$\nabla \ldots = \frac{\vec{p}}{r^3} - \frac{3\vec{r}}{r^5}\, \vec{p}\cdot\vec{r}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$ DIV.

$\nabla \cdot$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \ldots \right)$$

ROT

$$\nabla \cdot \vec{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$$

$$\boxed{\text{DÚ: } \nabla \cdot \left(\frac{\vec{r}}{r^3} \right)}$$

$$\nabla \times \frac{\vec{r}}{r^3} \Big|_x = \left(\frac{\partial \frac{z}{r^3}}{\partial y} - \frac{\partial \frac{y}{r^3}}{\partial z} \right)$$

$$\frac{\partial}{\partial y} \frac{z}{(x^2+y^2+z^2)^{3/2}} - \frac{\partial}{\partial z} \frac{y}{(x^2+y^2+z^2)^{3/2}} =$$

$$= \frac{-z \frac{3}{2} \sqrt{x^2+y^2+z^2} \cdot 2y}{r^6} + \frac{3y \cdot z \cdot r}{r^6}.$$

$$= \frac{-3yz + 3yz}{r^5} = 0$$

$$\nabla \times \frac{\vec{r}}{r^3} = \vec{0} = (0, 0, 0)$$

$$\nabla\cdot\nabla T = \nabla\cdot\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right) = \frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}$$

$$\equiv \Delta T \qquad \text{LAPLACE}$$

$$\nabla^2 T$$

$$\nabla\times\nabla T = \nabla\times\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right)$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{vmatrix} = \Big|_x = \frac{\partial^2 T}{\partial y\,\partial z} - \frac{\partial^2 T}{\partial z\,\partial y}$$

$$= 0$$

$$\nabla\times\nabla = \vec{0}$$

$$\nabla\cdot\left(\nabla\times\vec{A}\right) = \nabla\cdot\left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y, \ldots\right)$$

$$= \cancel{\frac{\partial^2 A_z}{\partial x\,\partial y}} - \cancel{\frac{\partial^2 A_y}{\partial x\,\partial z}} + \cancel{\frac{\partial^2 A_x}{\partial y\,\partial z}} - \cancel{\frac{\partial^2 A_z}{\partial y\,\partial x}} +$$

$$+ \cancel{\frac{\partial^2 A_y}{\partial z\,\partial x}} - \cancel{\frac{\partial^2 A_x}{\partial z\,\partial y}} = 0$$

$$\nabla\cdot\left(\nabla\times\ \right) = 0$$

$$\nabla\times\nabla\times\vec{A} \qquad \vec{A}\times\vec{B}\times\vec{C} = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{B}\cdot\vec{A}\right)$$

$$= \nabla\left(\nabla\cdot\vec{A}\right) - \vec{A}\overbrace{\left(\nabla\cdot\nabla\right)}^{\Delta}$$

$$= \nabla\left(\nabla\cdot\vec{A}\right) - \Delta\vec{A}$$