English for Mathematicians, UNIcert® III

PART ONE

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READING COMPREHENSION

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Assignment Sheet

Candidate's Number:

Name:

Surname:

Date:

Candidate's Signature: ________________________________

90 minutes / 65 points

SUBTEST A

Read the following text.
NB: For the purpose of Task Three in SUBTEST A, some of the words are underlined.

Prime Number Conspiracy

In a recent paper published in 2016, Kannan Soundararajan and Robert Lemke Oliver of Stanford University have presented both numerical and theoretical evidence that prime numbers <u>repel</u> other would-be primes that end in the same digit, and have varied <u>predilections</u> for being followed by primes ending in the other possible final digits.

Among the first billion prime numbers, for instance, a prime ending in 9 is almost 65 percent more likely to be followed by a prime ending in 1 than another prime ending in 9.

This "conspiracy" among prime numbers seems, at first glance, to violate a longstanding assumption in number theory: that prime numbers behave much like random numbers. Most mathematicians would have assumed that a prime should have an equal chance of being followed by a prime ending in 1, 3, 7 or 9 (the four possible endings for all prime numbers except 2 and 5).

Yet the pair's work doesn't <u>upend</u> the notion that primes behave randomly so much as point to how subtle their particular mix of randomness and order is.

Soundararajan was drawn to study consecutive primes after hearing a lecture at Stanford by the mathematician Tadashi Tokieda, of the University of Cambridge, in which he mentioned a counterintuitive property of coin-tossing: If Alice tosses a coin until she sees a head followed by a tail, and Bob tosses a coin until he sees two heads in a row, then on average, Alice will require four tosses while Bob will require six tosses, even though head-tail and head-head have an equal chance of appearing after two coin tosses.

Soundararajan wondered if similarly strange phenomena appear in other contexts. Since he has studied the primes for decades, he turned to them — and found something even stranger than he had bargained for.

Looking at prime numbers written in base 3 — in which roughly half the primes end in 1 and half end in 2 — he found that among primes smaller than 1,000, a prime ending in 1 is more than twice as likely to be followed by a prime ending in 2 than by another prime ending in 1. Likewise, a prime ending in 2 prefers to be followed by a prime ending in 1.

Soundararajan then showed his findings to postdoctoral researcher Lemke Oliver, who was shocked. He immediately wrote a program that searched much farther out along the number line — through the first 400 billion primes. Lemke Oliver again found that primes seem to avoid being followed by another prime with the same final digit.

Lemke Oliver and Soundararajan discovered that this sort of bias in the final digits of consecutive primes holds not just in base 3, but also in base 10 and several other bases; they conjecture that it's true in every base. The biases that they found appear to even out, little by little, as you go farther along the number line — but they do so at a snail's pace.

Lemke Oliver and Soundararajan's first guess for why this bias occurs was a simple one: Maybe a prime ending in 3, say, is more likely to be followed by a prime ending in 7, 9 or 1 merely because it encounters numbers with those endings before it reaches another number ending in 3. For example, 43 is followed by 47, 49 and 51 before it hits 53, and one of those numbers, 47, is prime.

But the pair of mathematicians soon realized that this potential explanation couldn't account for the magnitude of the biases they found. Nor could it explain why, as the pair found, primes ending in 3 seem to like being followed by primes ending in 9 more than 1 or 7. To explain these and other preferences, Lemke Oliver and Soundararajan had to delve into the deepest model mathematicians have for random behavior in the primes.

Prime numbers, of course, are not really random at all — they are completely determined. Yet in many respects, they seem to behave like a list of random numbers, governed by just one overarching rule: The approximate density of primes near any number is inversely proportional to how many digits the number has.

The primes' preferences about the final digits of the primes that follow them can be explained, Soundararajan and Lemke Oliver found, using a much more refined model of randomness in primes, something called the prime k-tuples conjecture. Originally stated by mathematicians G. H. Hardy and J. E. Littlewood in 1923, the conjecture provides precise estimates of how often every possible constellation of primes with a given spacing pattern will appear. A wealth of numerical evidence supports the conjecture, but so far a proof has eluded mathematicians.

The prime k-tuples conjecture subsumes many of the most central open problems in prime numbers, such as the twin primes conjecture, which posits that there are infinitely many pairs of primes — such as 17 and 19 — that are only two apart. Most mathematicians believe the twin primes conjecture not so much because they keep finding more twin primes, but because the number of twin primes they've found fits so neatly with what the prime k-tuples conjecture predicts.

In a similar way, Soundararajan and Lemke Oliver have found that the biases they uncovered in consecutive primes come very close to what the prime k-tuples conjecture predicts. In other words, the most sophisticated conjecture mathematicians have about randomness in primes forces the primes to display strong biases.

At this early stage, mathematicians say, it's hard to know whether these biases are isolated peculiarities, or whether they have deep connections to other mathematical structures in the primes or elsewhere.

SUBTEST B

Read the following text.
NB: For the purpose of Task One in SUBTEST B, some of the words are underlined.

Space Curves

Most of the discussion of the last section can be adapted to apply to curves in space (sometimes called twisted curves).

To start with, we again get the tangent as the limiting position of the secant when one point of intersection moves into coincidence with the other. But the three-dimensional case differs from the case of plane curves by the fact that there are infinitely many perpendiculars to the tangent at the point of contact; these perpendiculars fill out a plane which is called the normal plane at the point of the curve.

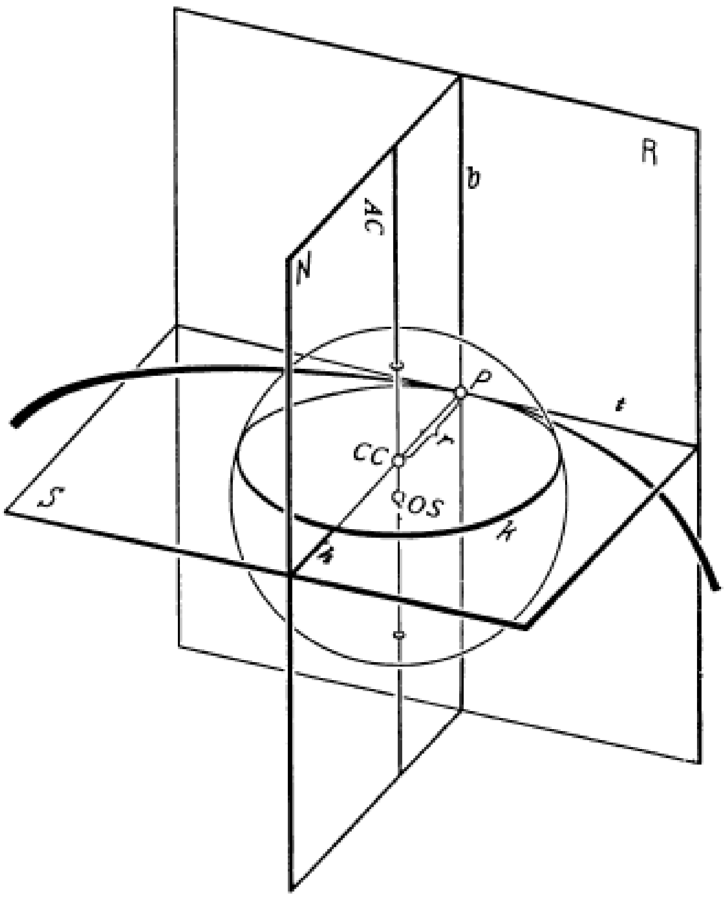

FIG. 189

P	the point under consideration
S	osculating plane
N	normal plane
R	rectifying plane
t	tangent
h	principal normal
b	binormal
k	circle of curvature
r	radius of curvature
AC	axis of curvature
CC	center of curvature
OS	center of osculating sphere

We shall try to find a plane lying as close to the curve as possible in the neighborhood of the point under consideration. To this end, we draw the plane passing through the tangent at the given point and through a neighboring point of the curve and let the second point move along the curve toward the point of contact of the tangent, which we hold fixed. In this process the plane approaches a limiting position. The limiting plane satisfies our requirement; it is called the *osculating plane* of the curve at the point under consideration.

Using a mode of expression introduced earlier, we say that the osculating plane has three coincident points in common with the curve. For this reason, the curve generally crosses its osculating plane at the point of contact, although it lies on one side of any other plane containing the tangent.

Since it contains the tangent, the osculating plane is perpendicular to the normal plane. Finally, let us consider that plane through the given point of the curve which is perpendicular both to the normal plane and to the osculating plane. It is called the *rectifying* plane.

The three planes just considered may be interpreted as coordinate planes in a three-dimensional Cartesian coordinate system which proves to be particularly well suited for describing the course of the curve at the point under consideration. One of the coordinate axes in this system is the tangent; the other two axes, which must lie in the normal plane, are called the *principal normal* and the *binormal*. The principal normal lies in the osculating plane, the binormal in the rectifying plane (see Fig. 189). This coordinate system, depending as it does on the point of the curve, is called the *moving trihedron* of the curve. It is the analogue of the coordinate system formed by the tangent and normal in the case of plane curves. In space, a coordinate system defines eight regions, called *octants*, as against four quadrants in the case of the plane. Thus the moving trihedron serves to distinguish eight types of points on a curve in much the same way as four types of points were distinguished, on page 174, for plane curves. Once again, only one of the cases is regular, and the others can occur only at isolated points (provided our curve is really a space curve, i.e. provided it does not lie wholly in a plane). At a regular point the curve intersects the osculating plane and the normal plane and remains on one side of the rectifying plane. We shall not discuss the other cases here. It may be mentioned, incidentally, that the twisted curves having a simple analytic structure, may, just like the plane curves, exhibit three additional types of singularities, namely double points, terminal points, and isolated points.

Let us generalize the Gaussian representation of plane curves to the case of three-dimensional curves. For this purpose, we use a sphere of unit radius. To every tangent of the curve (which we assume to be oriented, i.e., to have a definite sense of traversal), we draw the radius of the sphere parallel to the tangent and pointing in the same direction. Its extremity on the surface of the sphere is called the tangential image of the point on the curve. In this way the entire curve is represented by a definite curve on the sphere. If the principal normal or the binormal is used instead of the tangent, we get two more curves on the sphere. Referred to their respective moving trihedra, these three "spherical images" are connected with each other and with the original curve by certain simple relations. For example, the tangential indicatrix and the binormal indicatrix together characterize the eight above-mentioned types of point of a curve: the point on the original curve, the tangent, and the binormal may each either move on continuously or reverse its course, and the combinations of the various possibilities give us just those eight cases.

SUBTEST C

Read the definitions of mathematical notions and say which terms are defined.

Definition 1

____________________ is a set within which the values of a function lie. (Not only the set of values that the function actually takes.)

Definition 2

____________________ is a point x_0 at which f is differentiable and $f(x_0) = 0$.

Definition 3

____________________ is the determination of a set of divisors of a given integer, polynomial, etc., which, when multiplied together, give the original number, polynomial, etc.

Definition 4

____________________ is a point at which a function $f(x)$ has both left-hand and right-hand limits but the limits are not equal.

Definition 5

…

Definition 6

…

Definition 7

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Definition 8

…

Definition 9

…

Definition 10

…

SUBTEST D

Fill an appropriate phrase from the box in each gap. Use each phrase only once. There are three extra phrases that do not fit any gap.

framework	required	dependence	represent	made up	a point
include	the average	normalised	combine	calculate	interpolating
equal	properties	similarity	using	unit magnitude	restrict

Use of quaternions to represent transformations in 3D

The main practical application of this interesting algebra is to represent 3D rotations.

In fact quaternions can (1) __________ 3D reflections, rotations and scaling, however a single quaternion operation cannot (2) __________ translations so if we want to rotate, reflect or scale around (3) __________ other than the origin, then we would have to handle the translation part separately. To (4) __________ the resulting point (Pout) when we translate the point (Pin) (5) __________ quaternions then we use the following equations:

For Reflection & scaling: Pout = q * Pin * q
For Rotation & scaling: Pout = q * Pin * conj(q)

The majority of applications involve pure rotations, for this we (6) __________ the quaternions to those with (7) __________ and we use only multiplications and not addition to represent a combination of different rotations. When quaternions are (8) __________ in this way, together with the multiplication operation to (9) __________ rotations, they form a mathematical group, in this case SU(2).

We can use this to do lots of operations which are (10) __________ in practical applications such as combining subsequent rotations (and equivalently orientations), (11) __________ between them, etc.

When quaternions are used in this way we can think of them as being similar to axis-angle except that real part is (12) __________ to cos(angle/2) and the complex part is (13) __________ of the axis vector times sin(angle/2). It is quite difficult to give a physical meaning to a quaternion, and many people find this (14) __________ to axis-angle as the most intuitive way to think about it, others may just prefer to think of quaternions as an interesting mathematical system which has the same (15) __________ as 3D rotations.