Dekódovanie

$u \xrightarrow{K} v \rightsquigarrow v' \xrightarrow{K'} u$ $\quad u, v$ konečné

$\phantom{u \xrightarrow{K} v \rightsquigarrow}\ \underset{r}{\parallel}$

- ML – maximum likelihood

$$\arg\max_{v} \Pr[r \mid v]$$

- MAP – max. aposteriori prob.

$$\arg\max_{v} \Pr[v \mid r]$$

→ zohľadňuje $\Pr[v]$

$$\underset{MAP}{\Pr[v \mid r]} = \frac{\overset{ML}{\Pr[r \mid v]} \cdot \Pr[v]}{\Pr[r]}$$

Ak $\Pr[v]$ je zhruba rovnaká pre všetky v, ML = MAP

Viterbiho algoritmus: ML dekódovanie

$r = r_0 \dots r_t$

$r_i = (r_0, \dots, r_{c-1})$

$$\arg\max_{v} \Pr[r \mid v] = \arg\max_{v} \prod_{i}^{t} \Pr[r_i \mid v_i] = \arg\max_{v} \prod_{i}^{t} \prod_{j}^{c-1} \Pr[r_i^{(j)} \mid v_i^{(j)}] =$$

(znaky)

Predpoklad: pr. chýb na jednotlivých v_i sú nezávislé

predpoklad

$$= \arg\max_{v} \sum_{i=0}^{t} \sum_{j=0}^{c} \log \Pr[r_i^{(j)} \mid v_i^{(j)}] \;\textcircled{=}$$

$$\mu(r_i^{(j)}, v_i^{(j)}) = a \cdot \left(\log \Pr[r_i^{(j)}, v_i^{(j)}] - f(r_i^{(j)})\right) \qquad a > 0$$

$$\textcircled{=}\; \arg\max_{v} \sum_{i=0}^{t} \sum_{j=0}^{c-1} \mu(r_i^{(j)}, v_i^{(j)})$$

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1. Hard decoding

$\overset{v}{01100\dots1} \rightsquigarrow \overset{r}{10110\dots1}$

Predpoklad:
- $0 \rightsquigarrow 1$, $1 \rightsquigarrow 0$ majú rovnakú pravdep.
- $1 \rightsquigarrow 1$, $0 \rightsquigarrow 0$

$\Pr[b \mid 1-b] = \varepsilon \dots$ pr. chyby $\qquad$ BÚNO: $\varepsilon < 1/2$

$\Pr[b \mid b] = 1 - \varepsilon$

$$\Pr[a \mid b] = (1-\varepsilon) \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right)^{\mathbb{1}_{a \neq b}}$$

$$\mu(s,t) = a \cdot \left(\log\left((1-\varepsilon)\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\mathbb{1}_{s\neq t}}\right) - f(s)\right)$$

$$= a \log\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\mathbb{1}_{s\neq t}} + a \log(1-\varepsilon) - a \cdot f(s)$$

$$a = -\frac{1}{\log\left(\frac{\varepsilon}{1-\varepsilon}\right)} \quad ; \qquad f(s) = \log \varepsilon$$

$$\mu(b,b) = -\frac{1}{\log\left(\frac{\varepsilon}{1-\varepsilon}\right)} \cdot \log(1-\varepsilon) + \frac{1}{\log\left(\frac{\varepsilon}{1-\varepsilon}\right)} \cdot \log \varepsilon =$$

$$= \frac{1}{\log(-)} \left(\log \varepsilon - \log(1-\varepsilon)\right) = 1$$

$$\mu(a,b) = \underbrace{-\frac{1}{\log\left(\frac{\varepsilon}{1-\varepsilon}\right)} \cdot \log\frac{\varepsilon}{1-\varepsilon}}_{-1} \underbrace{- \frac{1}{\log\left(\frac{\varepsilon}{1-\varepsilon}\right)} \cdot \log(1-\varepsilon) + \frac{1}{\log\left(\frac{\varepsilon}{1-\varepsilon}\right)} \log\varepsilon}_{1}$$

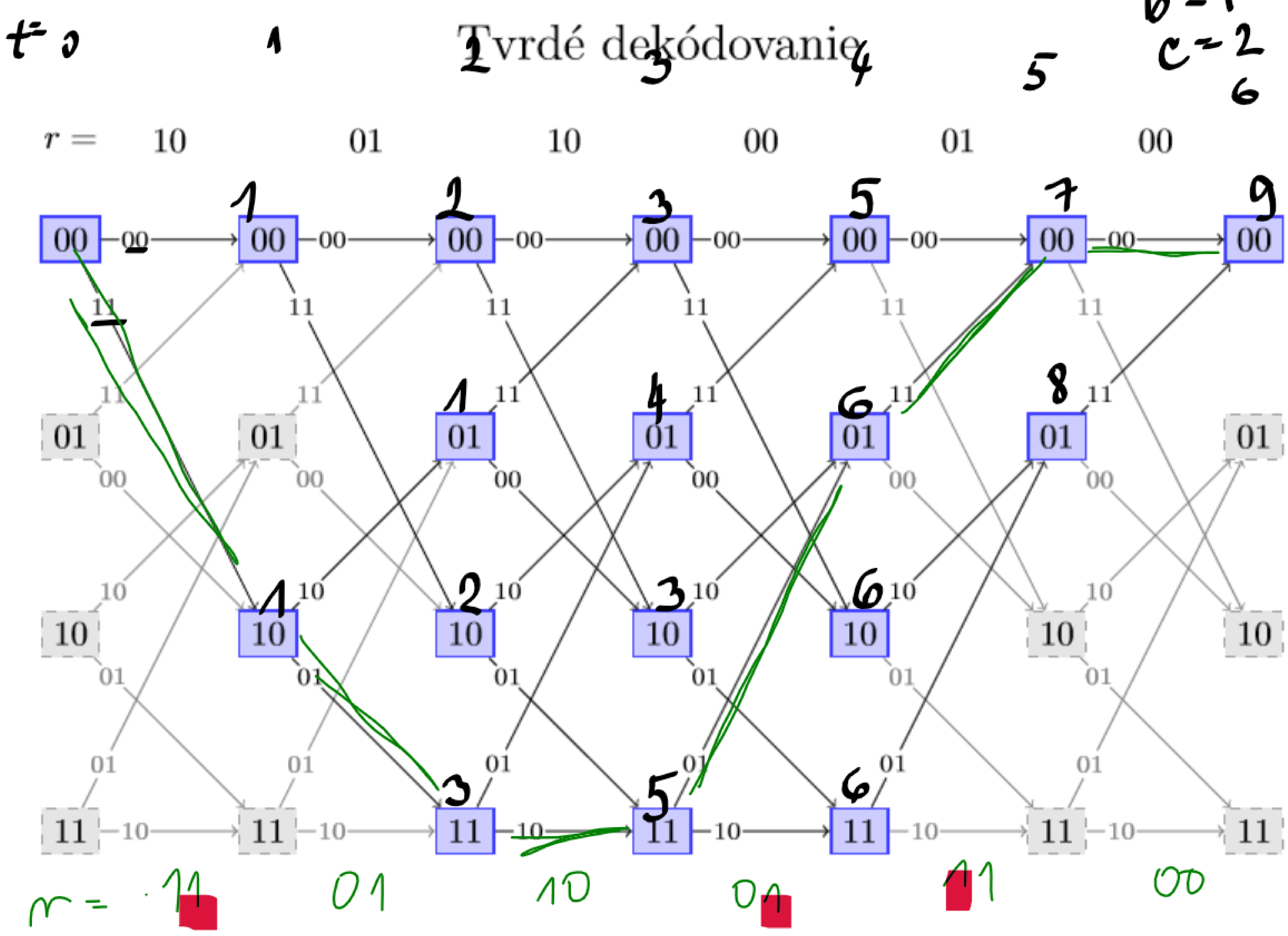
$$= 0$$

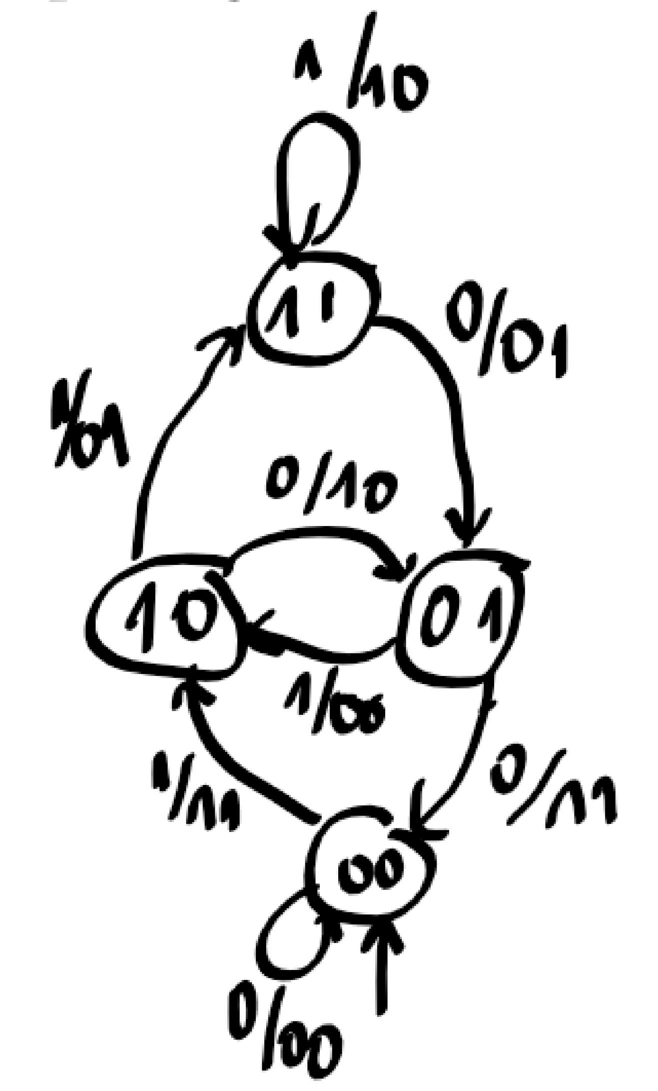
	b
b	1
1-b	0

$b = 1$
$c = 2$

$G = (1+D+D^2, 1+D^2)$

Tvrdé dekódovanie



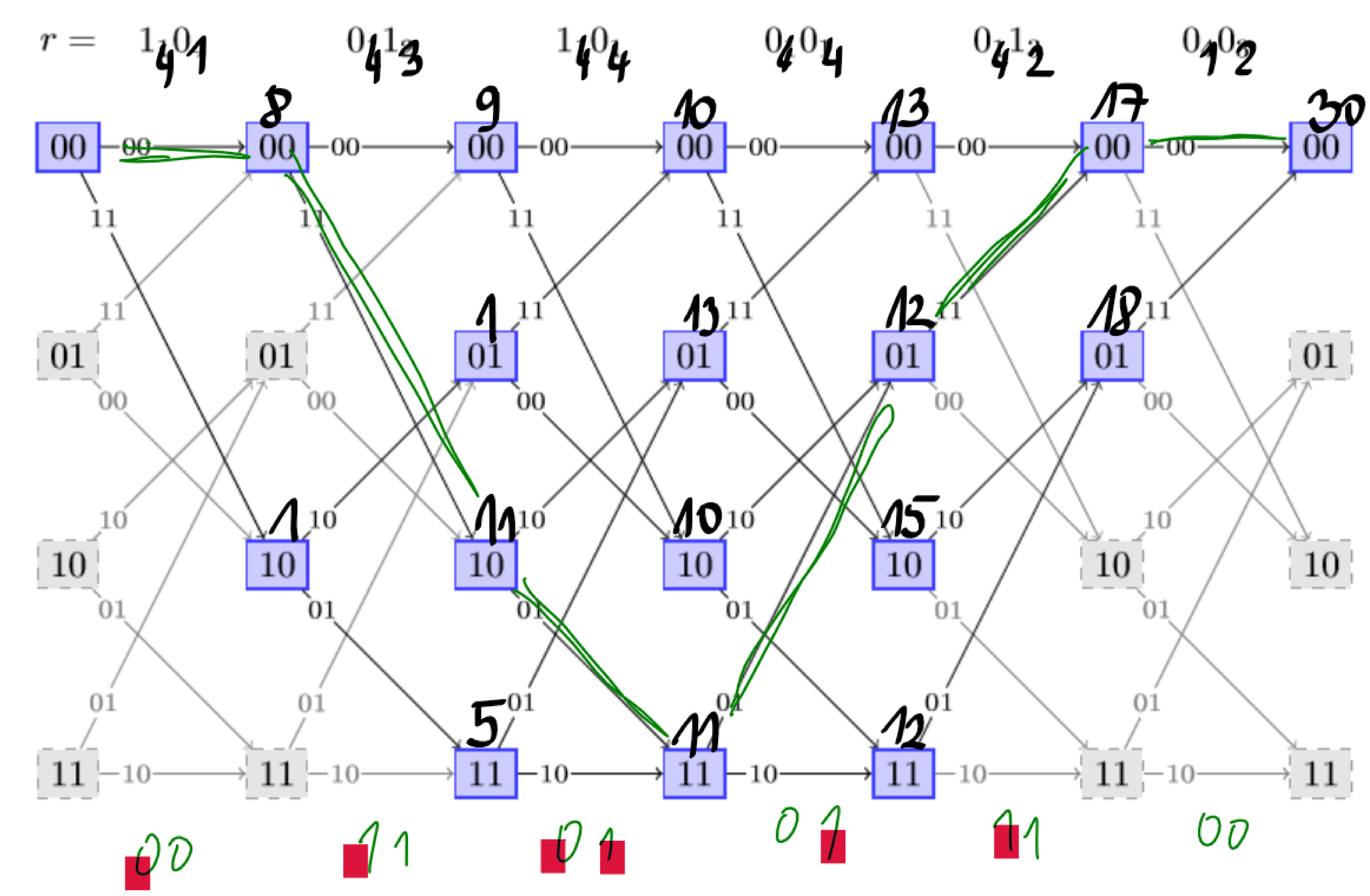


$m = 111000$

$1+D+D^2$

Mäkké dekódovanie

$\mu(b_i, b)$	b_1	b_2	b_3	b_4	$(1-b)_1$	$(1-b)_2$	$(1-b)_3$	$(1-b)_4$
b	8	5	3	1	0	0	0	0



$m = 011000$

Soft decoding: $0,1 \rightsquigarrow 0_1, 0_2, 0_3, 0_4, 1_1, 1_2, \dots$

Prijaté symboly b_1, b_2, b_3, b_4 ← sme si menej istí...

↖ sme si dosť istí, že je to b

$Pr[b_i \mid b]$	b_1	b_2	b_3	b_4
b	0,434	0,197	0,167	0,111
$1-b$	0,002	0,008	0,023	0,058

$$\Big\downarrow \mu$$

	b_1	b_2	b_3	b_4
b	7,7	4,6	2,8	0,9
$1-b$	0	0	0	0

$\longrightarrow$

	b_1	b_2	b_3	b_4
b	8	5	3	1
$1-b$	0	0	0	0

$$(1+D+D^2)\left(1+D+D^2,\ 1+D^2\right) = \left(1+D^2+D^4,\ 1+D+D^3+D^4\right)$$

$$\Big\downarrow$$

$$(\underline{101010\ },\ \underline{11011}0)$$

$$r = (11, 01, 10, 01, 11, 01)$$

Problém: Zložitosť Vit. algoritmu je pol. v #stavov, #stavov je exponenciálny voči 'dĺžke registra'

⇒ môže byť exponenciálna k dĺžke vstupu

r má dĺžku t.c?