$$\lim_{n\to\infty} \int_0^1 \frac{n\sin(x)}{1+n^2\sqrt{x}} \overset{\text{Lebesgue}}{=} 0.$$

$$\lim_{n\to\infty} \frac{n\sin(x)}{1+n^2\sqrt{x}} = \lim_{n\to\infty} \frac{\frac{\sin(x)}{n}}{\frac{1}{n^2}+\sqrt{x}} \overset{AL}{=} \frac{0}{0+\sqrt{x}} = 0.$$

$$\left|\frac{n\sin(x)}{1+n^2\sqrt{x}}\right| \le \frac{n}{1+n^2\sqrt{x}} \le \frac{n}{n^2\sqrt{x}} \le \frac{1}{\sqrt{x}}, \quad \int_0^1 \frac{1}{\sqrt{x}}\,dx < \infty.$$