22.12. – místnost 101

22/4d: $f(x) = \sin x + \operatorname{tg} x - x$ na $(0, \frac{\pi}{2})$

$f(0) = f(0+) = 0$

$f(\frac{\pi}{2}-) = +\infty$

f je spojitá na $(0, \frac{\pi}{2})$

$x \in (0, \frac{\pi}{2})$: $f'(x) = \underbrace{\cos x}_{>0} + \underbrace{\frac{1}{\cos^2 x} - 1}_{>0} > 0$

$\Rightarrow$ f je na $(0, \frac{\pi}{2})$ rostoucí, + f spojitá v $[0)$ } f rostoucí na $[0, \frac{\pi}{2})$

$\Rightarrow \forall x \in (0, \frac{\pi}{2}): f(x) > f(0) = 0$

22/5a: $x > y$, Lagrangeova věta:

$\sin x - \sin y = \sin'(\xi)(x-y)$ pro $\xi \in (y, x)$

$|\sin x - \sin y| \le |\cos(\xi)|\,|x-y| \le |x-y|$

$K(x) = 1-2x^2-(1-x^2)e^{-x^2}$

? $K<0$ na $(0,+\infty)$?

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$K(0)=0$; K spojitá na $[0,+\infty)$

$$K'(x) = -4x - e^{-x^2}\left(-2x+(1-x^2)(-2x)\right) =$$

$$= -2x\left(2-e^{-x^2}(2-x^2)\right)$$

$$= -2x\left(\underbrace{2-2e^{-x^2}}_{>0} + x^2e^{-x^2}\right) < 0 \qquad x>0$$

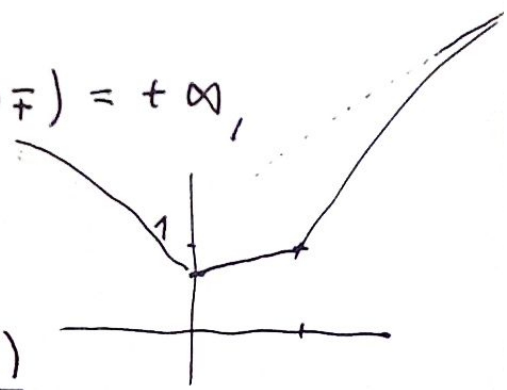
$x>0$

$\Rightarrow$ K je klesající na $[0,+\infty)$ $\Rightarrow$ $\forall x>0:\ K(x)<K(0)=0.$

22/1 $\quad f(x) := |x| + \operatorname{arctg}(|x-1|)$

1) $D(f)=\mathbb{R}$, f je spojitá na $\mathbb{R}$, $f(\pm\infty\mp)=+\infty$,

$f(0) = \frac{\pi}{4}$; $f(1)=1$



2) $x\notin\{0,1\}$: $f'(x) = \operatorname{sgn} x + \dfrac{\operatorname{sgn}(x-1)}{1+(x-1)^2}$

$x\in(-\infty,0)$: $f'(x) = -1+\dfrac{-1}{1+(x-1)^2} = \dfrac{-1-x^2+2x-1-1}{1+(x-1)^2}$

$$= \frac{-x^2+2x-3}{x^2-2x+2} < 0$$

$D = 4-4\cdot 3<0$

$\Rightarrow$ f je klesající na $(-\infty,0]$

$x\in(0,1):\ f'(x) = 1+\frac{-1}{x^2-2x+2} = \frac{x^2-2x+1}{x^2-2x+2} = \frac{(x-1)^2}{x^2-2x+2} > 0$

f je rostoucí na $[0,1]$

$x\in(1,+\infty):\ f'(x) = 1+\frac{1}{x^2-2x+2} = \frac{x^2-2x+3}{x^2-2x+2} > 0$

f je rostoucí na $[1,+\infty)$

$\Rightarrow$ • f má glob. minimum v 0 ; $f(0) = \frac{\pi}{4}$

• nemá glob. maximum

• $\mathcal{H}(f) = \left[\frac{\pi}{4}, +\infty\right)$

3) $x\notin\{0,1\}:\ f''(x) = \overbrace{\mathrm{sgn}(x-1)(-1)\frac{2x-2}{(x^2-2x+2)^2}}^{2|x-1|}$

$\Rightarrow f''<0 \Rightarrow f$ je konkávní na $(-\infty,0]$

a na $[0,1]$ a na $[1,+\infty)$

4) Doplnění: $f'_+(0) \underset{\uparrow}{=} f'(0+) = \frac{1}{2}$, $f'_-(0) = -\frac{3}{2}$

f spojitá v 0

$f'_+(1) = 2$; $f'_-(1) = 0$

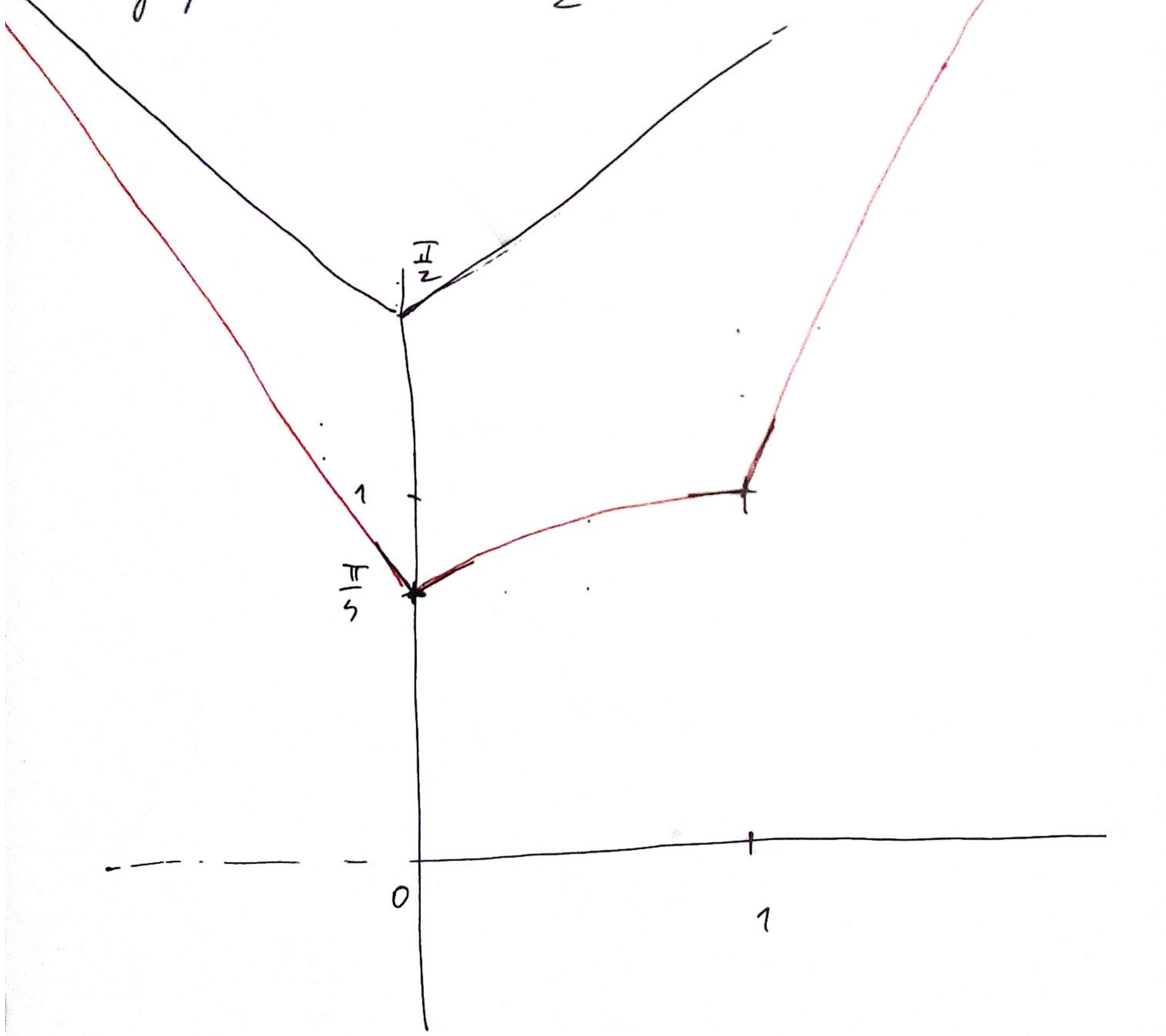
asymptoty: $\lim\limits_{x\to+\infty}\frac{f(x)}{x} = \lim\limits_{x\to+\infty}\frac{x}{x} + \frac{\operatorname{arccotg}(x-1)}{x} = 1$

$\lim\limits_{x\to-\infty}\frac{f(x)}{x} = \lim\limits_{x\to-\infty}\frac{-x}{x} + \frac{\operatorname{arccotg}(1-x)}{x} = -1$

$$\lim_{x\to+\infty} f(x) - x = \lim_{x\to+\infty} \operatorname{arctg}(x-1) = \frac{\pi}{2}$$

$$\lim_{x\to-\infty} f(x) - (-x) = \lim_{x\to-\infty} \operatorname{arctg}(1-x) = \frac{\pi}{2}$$

Asintoto a $+\infty$: $x+\frac{\pi}{2}$ a $-\infty$: $-x+\frac{\pi}{2}$



$$f(x) = \begin{cases} 0 & \text{na } (-1,1)^c \\ e^{-\frac{1}{1-x^2}} & \text{na } (-1,1) \end{cases}$$

$D(f) = \mathbb{R}$; f je spojitá v $\mathbb{R} \setminus \{\pm 1\}$, navíc

$$f(1+) = 0, \quad \lim_{x \to 1-} f(x) = 0 \Rightarrow f \text{ spojitá v } \pm 1$$

$$f'(x) \underset{\underset{(-1,1)}{\uparrow}}{=} -e^{-\frac{1}{1-x^2}} (-1) \frac{-2x}{(1-x^2)^2} = -e^{-\frac{1}{1-x^2}} \frac{2x}{(1-x^2)^2}$$

$$f'_-(1) = \lim_{x \to 1-} f'(x) = \lim_{x \to 1-} -e^{-\frac{1}{1-x^2}} \frac{2x}{(1-x^2)^2}$$

$$\overset{AL}{=} -2 \lim_{x \to 1-} e^{-\frac{1}{1-x^2}} \cdot \frac{1}{(1-x^2)^2} = 0 = f'_+(1)$$

vnější: $e^{-y} y^2 \xrightarrow{y \to +\infty} 0$

vnitřní: $\frac{1}{1-x^2} \xrightarrow{x \to 1-} +\infty$

$$\Rightarrow f'(1) \overset{+\text{podm } P}{=} f'(-1) = 0$$

Dú: $\forall k \in \mathbb{N}$: $f^{(k)}$ je def na $\mathbb{R}$ a spojitá, $f^{(k)}(1) = 0$

Pozn: $T_n^{f,1}(x) = 0$; Spec pro $x \in (-1,1)$:

$n \in \mathbb{N}$: $\quad \lim_{n \to +\infty} T_n^{f,1}(x) \neq f(x)$