1) $\lim_{n\to\infty} \left(\sqrt[3]{n^3+\sqrt{n}} - \sqrt[3]{n^3-1}\right)\cdot\sqrt{3n^3+1} =$ $\quad a^3-b^3=(a-b)\cdot(a^2+ab+b^2)$ $\quad \boxed{26-1}$

$$= \lim_{n\to\infty} \left(\sqrt[3]{n^3+\sqrt{n}} - \sqrt[3]{n^3-1}\right)\cdot \frac{\left(\sqrt[3]{n^3+\sqrt{n}}\right)^2 + \sqrt[3]{n^3+\sqrt{n}}\cdot\sqrt[3]{n^3-1} + \left(\sqrt[3]{n^3-1}\right)^2}{-\ //\ -}\sqrt{3n^3+1} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n^3}+\sqrt{n}-\cancel{n^3}+1}{\left(\sqrt[3]{\quad}\right)^2+\sqrt[3]{\quad}\cdot\sqrt[3]{\quad}+\left(\sqrt[3]{\quad}\right)^2}\cdot\sqrt{3n^3+1}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \qquad \frac{1}{n^2} = \frac{1}{\sqrt{n}}\cdot\frac{1}{\sqrt{n^3}}$$

$$= \lim_{n\to\infty} \frac{1-\frac{1}{\sqrt{n}}}{\left(\sqrt[3]{1+\frac{1}{n^{\frac{5}{2}}}}\right)^2+\sqrt[3]{1+\frac{1}{n^{\frac{5}{2}}}}\cdot\sqrt[3]{1-\frac{1}{n^3}}+\left(\sqrt[3]{1-\frac{1}{n^3}}\right)^2}\cdot\sqrt{3+\frac{1}{n^3}}$$

$$\overset{\cancel{*}}{=} \frac{1}{1+1+1}\cdot\sqrt{3} = \sqrt{3}$$

$$\lim_{n\to\infty}\sqrt[3]{1+\frac{1}{n^{\frac{5}{2}}}} \overset{\text{Heine}}{=} \lim_{x\to\infty}\sqrt[3]{1+\frac{1}{x^{\frac{5}{2}}}} \overset{\text{VOLSF}}{\underset{(S)}{=}} \sqrt{1} = 1$$

(S) $\sqrt[3]{\ }$ spojitá v 1

analogicky $\lim_{n\to\infty}\sqrt[3]{1-\frac{1}{n^3}} = 1$ a $\lim_{n\to\infty}\sqrt{3+\frac{1}{n^3}} = \sqrt{3}$

Určete $a, b \in \mathbb{R}$

$$\lim_{x\to 0} \frac{\cos ax + x \cdot \arctan bx - b}{x^4} = \in \mathbb{R}.$$ Spočtěte jí

$$\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{4!} + o(y^4) \qquad \cos ax = 1 - \frac{(ax)^2}{2} + \frac{(ax)^4}{4!} + o(x^4)$$

$$\arctan y = y - \frac{y^3}{3} + o(y^3) \qquad \arctan bx = bx - \frac{(bx)^3}{3} + o(x^3)$$

$$x \cdot \arctan bx = bx^2 - \frac{b^3x^4}{3} + o(x^4)$$

$$= \lim_{x\to 0} \frac{1 - \frac{(ax)^2}{2} + \frac{(ax)^4}{4!} + o(x^4) + bx^2 - \frac{b^3x^4}{3} + o(x^4) - b}{x^4} =$$

musí být $1 - b = 0 \Rightarrow b = 1$

$$-\frac{(ax)^2}{2} + bx^2 = 0 \Rightarrow -\frac{a^2}{2} + \overset{b}{\overset{\shortparallel}{1}} = 0 \Rightarrow a = \sqrt{2}$$

$$= \frac{a^4}{4!} + 0 - \frac{b^3}{3} + 0 = \frac{4}{4!} - \frac{1}{3} = \frac{1}{6} - \frac{1}{3} = -\frac{1}{6}$$

$$\overset{\text{l'H}}{\underset{"0/0"}{=}} \begin{pmatrix} 1-b=0 \\ \Downarrow \\ b=1 \end{pmatrix} \lim_{x\to 0} \frac{(-\sin ax) \cdot a + \arctan x + x \cdot \frac{1}{1+x^2}}{4x^3} \overset{"0/0"}{=}$$

$$= \lim_{x\to 0} \frac{(-\cos ax)\cdot a^2 + \frac{1}{1+x^2} + \frac{1}{1+x^2} + x\cdot\frac{-1}{(1+x^2)^2}\cdot 2x}{12x^2} \overset{"0"}{\underset{"0"}{=}}$$

dozad $\frac{-a^2+1+1}{0} \Rightarrow a=\sqrt{2}$

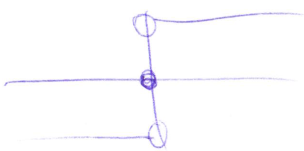
$$= \lim_{x\to 0} \frac{2-\cos(\sqrt{2}\cdot x)\cdot 2}{12x^2} + \lim_{x\to 0} \frac{-2+\frac{1}{1+x^2}+\frac{1}{1+x^2}-\frac{2x^2}{(1+x^2)^2}}{12x^2}$$

3) $\sqrt[3]{(x+2)^2} - \sqrt[3]{(x-2)^2}$

4) (i) $\lim_{x\to 0} f(x) = 1 \Rightarrow \lim_{x\to 0} f^2(x) = 1$

$$\lim_{x\to 0} f^2(x) \overset{?}{=} \lim_{x\to 0} f(x)\cdot \lim_{x\to 0} f(x) = 1\cdot 1 = 1$$

(ii) $\lim_{x\to 0} f^2(x) = 1 \Rightarrow \lim_{x\to 0} f(x) = 1$ NE

$f(x) = \operatorname{sgn} x$  $\lim_{x\to 0} \operatorname{sgn}^2 x = 1$ ale $\lim_{x\to 0} \operatorname{sgn} x$ [illegible]

(iii) $\lim_{x\to 0} f^3(x) = 1 \Rightarrow \lim_{x\to 0} f(x) = 1$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \sqrt[3]{f^3(x)} \underset{\text{VOLSF (1)}}{=} \sqrt[3]{\lim_{x\to 0} f^3(x)} = \sqrt[3]{1} = 1$$

$\sqrt[3]{y}$ je spojitá v 1.

Příklad: $\exists f: \mathbb{R} \to \mathbb{R}$ $\forall$ interval I $f(I) = \mathbb{R}$

Hrubá myšlenka $x \in \mathbb{R}$ $x = 110{,}\underbrace{01100111}$ ve 2-kové soustavě

$p_x(n)$ .. kolik je 1 v n cifrách

$p_x(1) = 0$ $p_x(2) = 1$, $p_x(3) = 2$ ...

Definujme
$$g(x) = \begin{cases} \lim_{n\to\infty} \frac{p_x(n)}{n}, & \text{pokud existuje a je } \neq 0 \text{ a } \neq 1 \\ \frac{1}{2} & \text{jinak} \end{cases}$$

Tvrdím, že g zobrazí libovolný interval na $(0, 1)$.

Pak $f(x) = \operatorname{tg}\left(\left(g(x) - \frac{1}{2}\right)\pi\right)$ zobrazí libovolný interval na $\mathbb{R}$.

Něco o LS:

- ŘADY $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $\sum_{n=0}^{\infty} a_n = \lim_{k\to\infty} \sum_{n=0}^{k} a_n$

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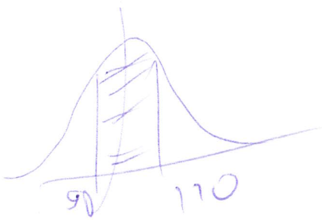
- INTEGRÁL $f' = g \Rightarrow \int g = f$

$\boxed{\int e^{-x^2}}$



- DIF. ROVNICE $f'(x) = 5 \cdot f(x)$

$f'(x) = a \cdot f(x) \Rightarrow f(x) = c_0 \cdot e^{ax}$

$|I| > \frac{1}{2^n} \cdot 100$

$11110,\overbrace{010001}^{n \text{ cifer}},$ až jsem zvolil jsem v I

čili $x \in I$ $\quad g(x) = \frac{3}{4}$ $\quad x = 11110,010001\underbrace{1110\,1110\,1110\,1110\,1110}\ldots \in I$

$\lim_{n\to\infty} \frac{p_x(n)}{n} = \frac{3}{4}$

$,000000000100000000001\ldots$

$g(x) = \frac{1}{10}$

$g(x) = \frac{1}{\sqrt{2}}$