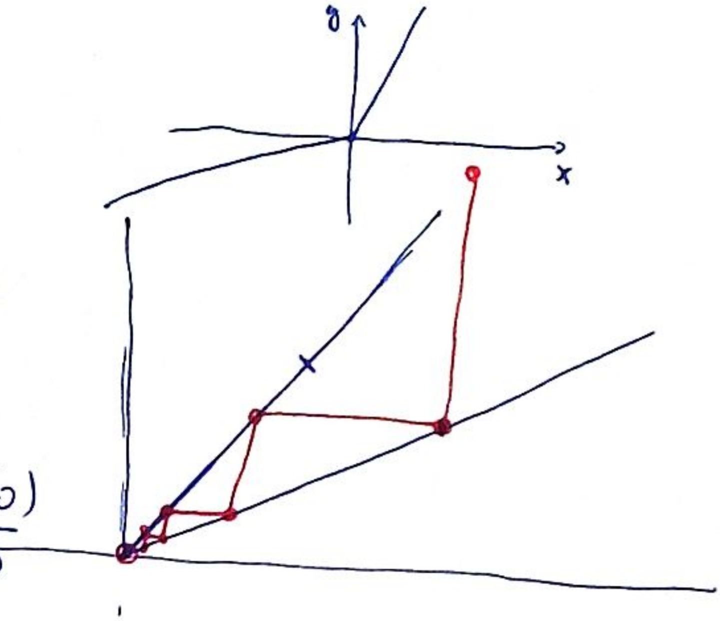
Cvičení 15.12. - místnost 101

19/8 $\quad +5 \quad -3$

/9 $\quad + \quad -8$

/10 $\quad +10 \quad -0$

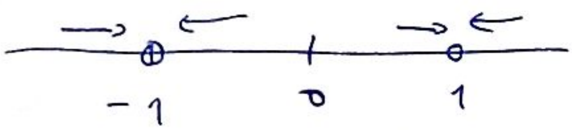
$f(x) = x^3 \; , \; f'(x) = 3x^2$



$$f'_+(0) = \lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0}$$

Průběh funkce

$$f(x) = \operatorname{arctg}\left(\frac{x^2+1}{x^2-1}\right)$$



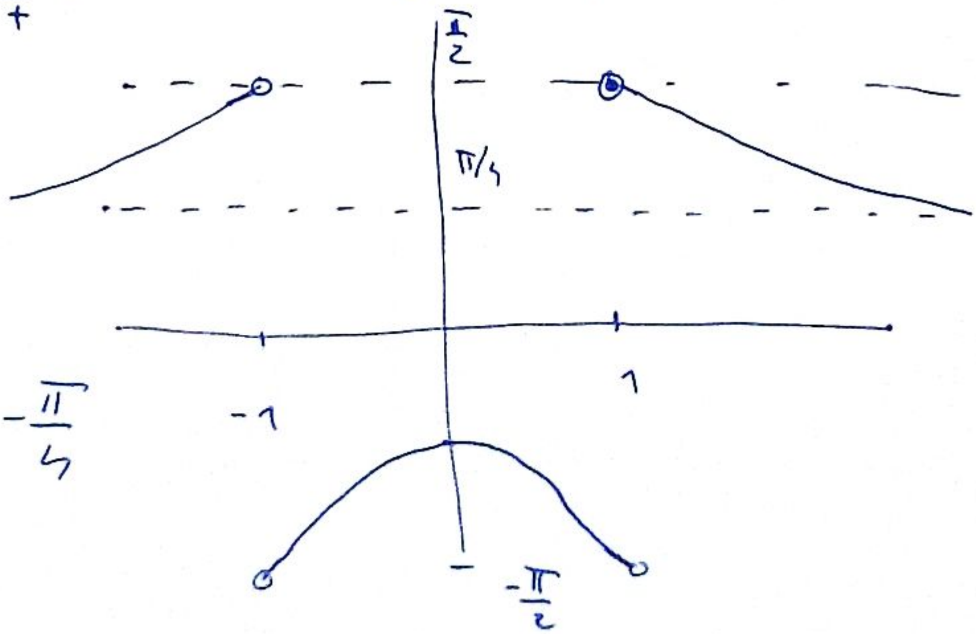
1) $\mathcal{D}(f) = \mathbb{R} \setminus \{\pm 1\}$; $f \in C(\mathcal{D}(f))$, f je sudá,

$f(\pm\infty \mp) := \lim_{x \to \pm\infty \mp} f(x) = \operatorname{arctg} 1 = \frac{\pi}{4}$

$f(1\pm) = \pm\frac{\pi}{2}$

$f(-1\pm) = \mp\frac{\pi}{2}$

$f(0) = \operatorname{arctg}(-1) = -\frac{\pi}{4}$



2) $x \in \mathcal{D}(f)$: $f'(x) = \dfrac{1}{1+\left(\dfrac{x^2+1}{x^2-1}\right)^2} \cdot \dfrac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2} =$

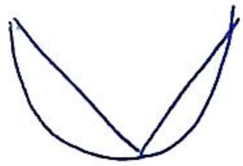
$$= \frac{-4x}{(x^2-1)^2+(x^2+1)^2} = \frac{-4x}{2x^4+2} = \frac{-2x}{x^4+1}$$

Problematické body: $-\infty \quad -1 \quad 0 \quad 1 \quad +\infty$

monotonie f: $\nearrow$ $(-\infty, -1)$, $\nearrow$ $(-1, 0]$, $\searrow$ $[0, 1)$, $\searrow$ $(1, +\infty)$

V 0 je lokální maximum.

$\mathcal{H}(f) = \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right]$



3) $x \in \mathcal{D}(f)$: $f''(x) = \dfrac{-2(x^4+1)+2x(4x^3)}{(x^4+1)^2} =$

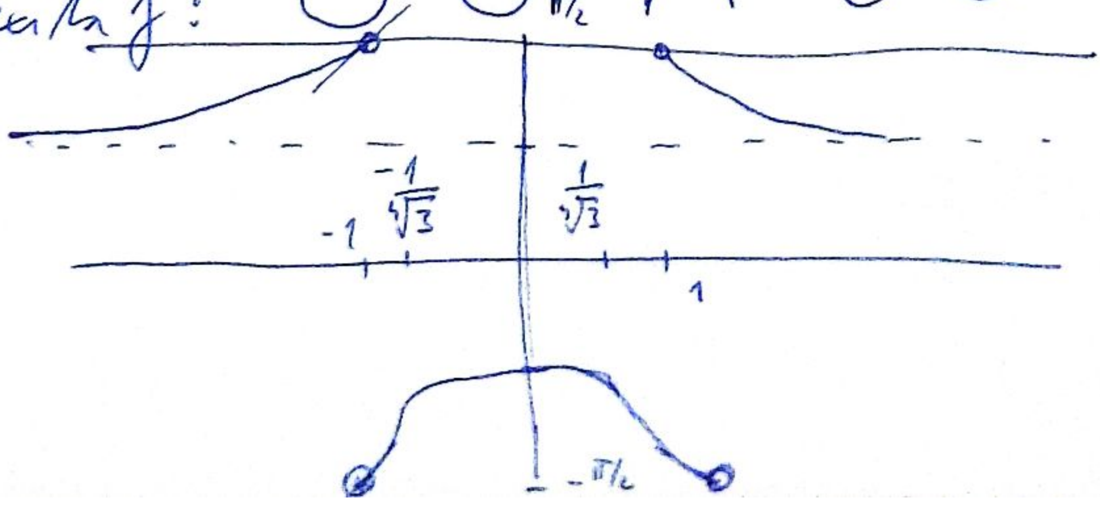
$$= \frac{6x^4-2}{(x^4+1)^2} = 2\,\frac{3x^4-1}{(x^4+1)^2}, \quad x = \frac{\pm 1}{\sqrt[4]{3}}$$

Problematické body: $-\infty \quad -1 \quad -\frac{1}{\sqrt[4]{3}} \quad +\frac{1}{\sqrt[4]{3}} \quad 1 \quad +\infty$

konvexita f: $\cup \quad \cup \quad \cap \quad \cup \quad \cup$

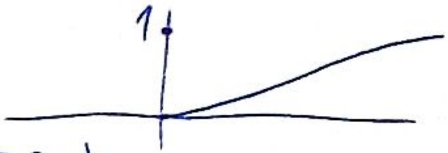
$f'(+1\pm) = -1$

$f'(-1\pm) = 1$



22/1: 1) $f(x) = \sqrt{1 - e^{-x^2}}$ $\mathcal{D}(f) = \mathbb{R}$, f sudá

$f(\pm\infty \mp) = 1$, $f(0) = 0$



$f \in C(\mathcal{D}(f))$

2) $x \neq 0$: $f'(x) = \dfrac{1}{\cancel{2}\sqrt{1-e^{-x^2}}} \cdot (+e^{-x^2})(+\cancel{2}x) =$

$$= \frac{x\, e^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

$x < 0 \Rightarrow$

$x = -\sqrt{x^2}$

Podezřelé body: $-\infty$ 0 $+\infty$

monotonie: $\searrow$ $\nearrow$

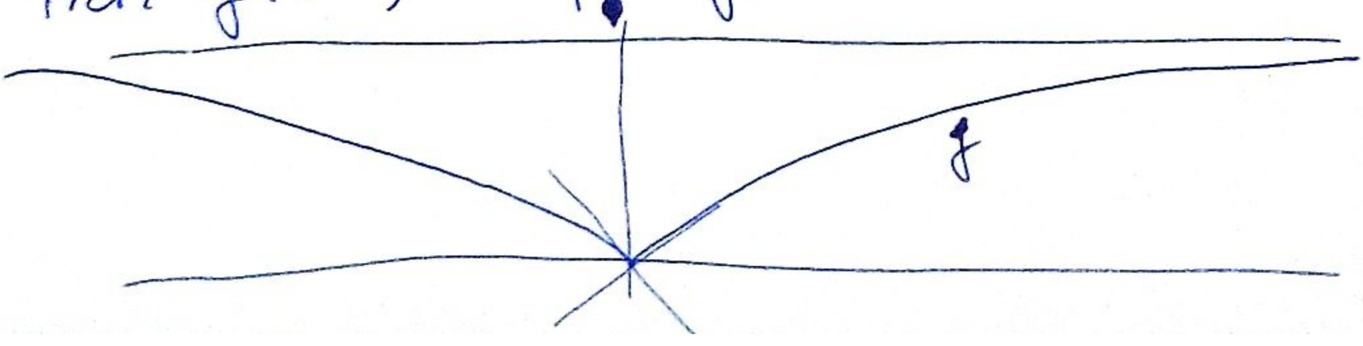
$\Rightarrow$ globální minimum v 0, glob. max. neex.

$\Rightarrow \mathcal{H}(f) = [0, 1)$

$$f'(0+) = f'_+(0) = \lim_{x\to 0+} e^{-x^2} \sqrt{\frac{-(-x^2)}{1-e^{-x^2}}} =$$

$$= \lim_{x\to 0+} e^{-x^2} \sqrt{\frac{-x^2}{e^{-x^2}-1}} = 1$$

Proto $f'(0-) = -1 = f'_-(0)$



3) $x \neq 0$: $f''(x) = \dfrac{e^{-x^2}\left(1+(-2x^2)\right)\sqrt{1-e^{-x^2}} - x e^{-x^2}\dfrac{x e^{-x^2}}{\sqrt{\cdots}}}{1-e^{-x^2}} =$

$$= \frac{1}{(1-e^{-x^2})^{3/2}}\left(e^{-x^2}(1-2x^2)(1-e^{-x^2}) - x^2 e^{-2x^2}\right)$$

$$= \frac{e^{-x^2}}{(1-e^{-x^2})^{3/2}}\left(1-2x^2 - e^{-x^2}(1-2x^2+x^2)\right)$$

$$= \frac{e^{-x^2}}{(1-e^{-x^2})^{3/2}}\left(1-2x^2-(1-x^2)e^{-x^2}\right).$$

$\Longrightarrow$ Průběh pro $K(x) := 1-2x^2-(1-x^2)e^{-x^2}$.

$\longrightarrow$ Rozmyslet do ~~[illegible]~~ úterý 22.12.

(ukázat, že $K < 0$ na $(0,+\infty)$)