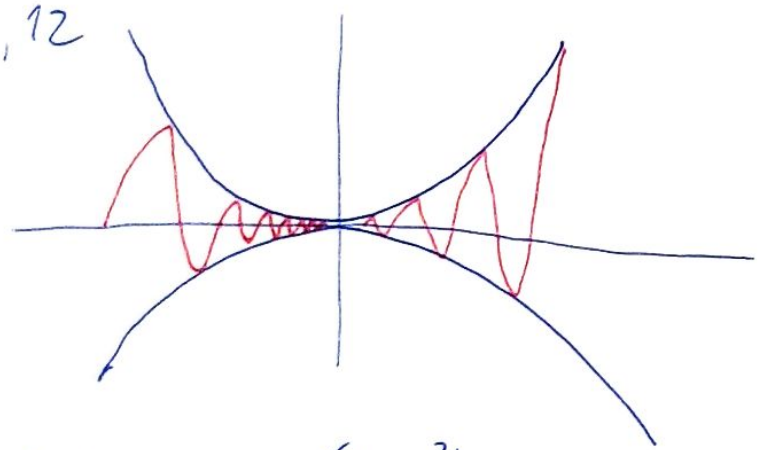
8.12.

Př: $\lim_{x\to 0} \frac{x+2x}{3x+4x} = \begin{cases} \cancel{1/3} \\ \cancel{1/2} \\ \cancel{3/7} \end{cases}$!

$\cancel{= \lim_{x\to 0} \frac{x + 2x}{3x+4x} = \lim_{x\to 0} \frac{x}{3x} = \frac{1}{3}}$ 0

18/11, 12



$f(x) = x^2 \sin\left(\left(\frac{1}{x}\right)^\alpha\right)$

pro vhodné α.

18/6 $f(x) = \arccos(1-x^2)$

$-1 \le 1-x^2 \le 1$; $x^2 \le 2$; $x \in [-\sqrt{2}, \sqrt{2}] = \mathcal{D}(f)$

Kde lze derivovat? Problém s $\arccos'(\pm 1) = -\infty$!

$\to$ vzorec pro der. sl. fce nelze použít v $x=0$.

$$\left.\begin{array}{l} x \ne 0 \\ x \ne \pm\sqrt{2} \\ x \in \mathcal{D}(f) \end{array}\right\}: f'(x) = \frac{-1}{\sqrt{1-(1-x^2)^2}} \cdot (-2x) = \cancel{\frac{2x}{\sqrt{x^2(2-x^2)}}}$$

$$= \frac{2x}{\sqrt{x^2(2-x^2)}} = \frac{2\,\mathrm{sgn}\,x}{\sqrt{2-x^2}}$$

$\sqrt{x^2} = |x|$

Podle věty 4.11:

$$\lim_{x\to 0+} f'(x) = \sqrt{2}, \quad f \text{ je v } 0 \text{ spojitá}$$

$$\overset{V4.11}{\Longrightarrow} \quad f'_+(0) = \sqrt{2}$$

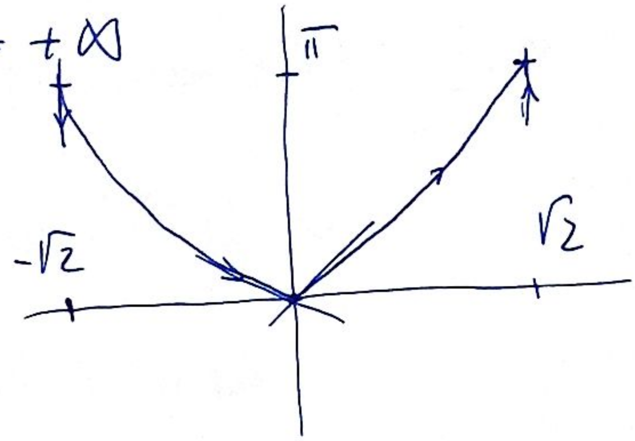
Podobně: $f'_-(0) = -\sqrt{2}$

$\Rightarrow f'(0)$ neexistuje

$$\lim_{x\to \sqrt{2}-} f'(x) = +\infty; \quad f \text{ je v } \sqrt{2} \text{ spojitá } \cancel{\text{zprava}} \text{ zleva}$$

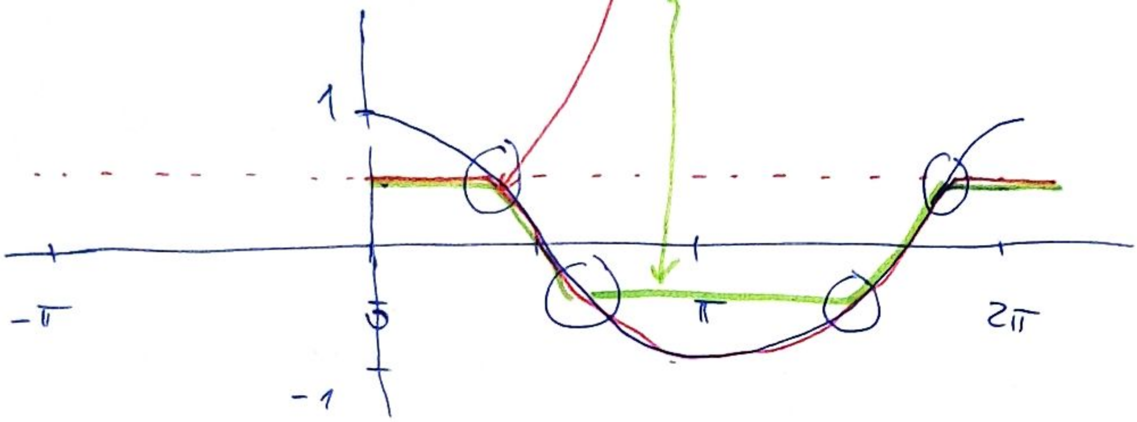
$$\Rightarrow \quad f'_-(\sqrt{2}) = +\infty$$

Podobně $f'_+(-\sqrt{2}) = -\infty$



K 19/2:

$$f(x) := \max\left(\min\left(\cos x, \frac{1}{2}\right), -\frac{1}{2}\right)$$



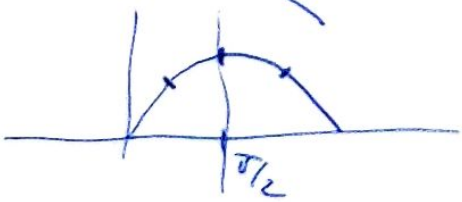
19/3 $\quad f(x)$ není def. v $x = -\frac{\pi}{4}$

$\Rightarrow f'_{\pm}\left(-\frac{\pi}{4}\right)$ neex.

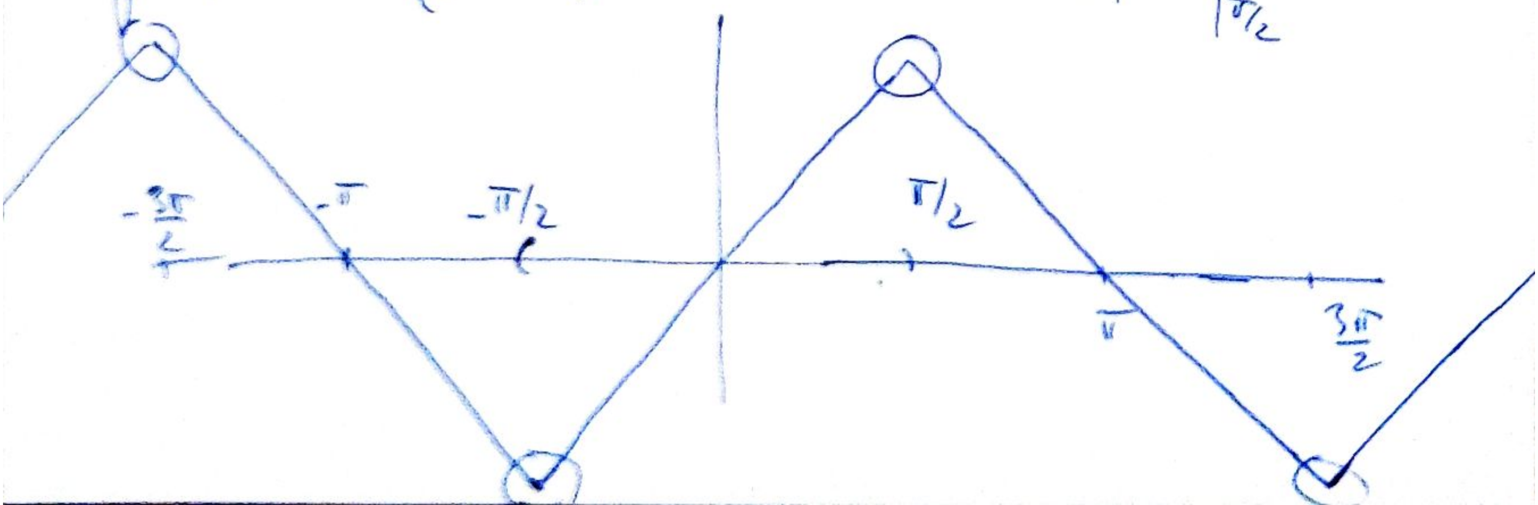
lze spočítat $f'\left(-\frac{\pi}{4}\pm\right) = \lim\limits_{x \to -\frac{\pi}{4}\pm} f'(x)$

19/8 $\quad \boxed{\text{Neplatí!}}$ $\quad \cancel{\forall x \in \mathbb{R} : \sqrt{x^2} = x}$!

$\cancel{\forall x \in \mathbb{R} : \arcsin(\sin x) = x}$!



$f(x) = \arcsin(\sin x)$



19/2

$$\lim_{x \to \frac{\pi}{3}^+} f'(x) = \lim_{x \to \frac{\pi}{3}^+} -\sin x = -\frac{\sqrt{3}}{2}$$

$$\lim_{x \to \frac{\pi}{3}^-} f'(x) = \lim_{x \to \frac{\pi}{3}^-} 0 = 0$$